



**THIRD SPACE
LEARNING**

Math Intervention Pack

Informally fitting bivariate data
with a line of best fit

Grade 8

How To Use This Resource

1. Title Slide

Use this slide to activate prior knowledge needed for lesson. Students should be encouraged to initially attempt the question presented independently.

2. Prior Learning

Use this slide to review the knowledge that will be required to be successful in this lesson. If students feel confident on the prior learning section of the Title Slide then this slide can be skipped

3. Let's Learn

Use this slide to introduce the concept. Tutors should work with the student to explore the concept together, usually using diagrams to support understanding.

4. Follow Me + Your Turn

The tutor should work through the follow me slide, modeling the process and explaining their thinking out loud.

Students should use the your turn slide as an opportunity to work through a question similar to the follow me questions. They should apply the method modeled by the tutor in the follow me slide. Students should be encouraged to explain their thinking out loud.

5. You Do

Students should work through a range of questions that build in complexity.

Tutors can offer support but students should initially be encouraged to attempt these questions independently.

6. Go Further

Use this slide to allow students to apply their understanding to a more challenging question in an unfamiliar context.

How To Use This Resource

7. Support for Slides

The support slide is used to support students during the lesson. In the tutor notes, there will be guidance as to when to use the support slide.

8. Check Your Understanding

Tutors should use this slide to assess the student's knowledge and whether or not they have mastered the concept within the lesson.

Standard

8.SP.A.2 - Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Key Mathematical Ideas

1. Draw an approximated line of best fit through data on a scatter plot.
2. Identify the equation, in the form $y = mx + b$, of the approximated line of best fit.
3. Interpret the slope of the line within the context of the data.

Overview

Terminology:

- **Line of best fit:** An equation of a line that estimates the relationship between variables.
- **Association:** Two variables that can be defined by a relationship.
- **Linear:** A relationship whose graph is a straight line.
- **Outlier:** A number in a data set that is significantly smaller or larger than the other data.
- **Inverse Operation:** An operation that undoes or reverses the effect of another operation (e.g., addition and subtraction are inverse operations).
- **Isolate:** To rearrange an equation to have the variable on one side and constants on the other side.
- **Slope:** The measure of a line's steepness as the change in y over the change in x .
- **y – intercept:** A point where a graph crosses the y -axis.

Sentence stems:

- This is an approximated line of best fit, because...
- Two points on the line are...
- We can find the linear equation by...
- The rate of change represents...

Overview

Common Misconceptions

Common Misconceptions	Tutoring Strategies	Checks for Understanding
Students who have trouble identifying points on the line.	Model how to find the easiest points on the line - ones that cross gridlines. Also remind students that this skill is about approximating, and ways to create the exact line of best fit will be explored later.	Ask students to look at the entire drawn line and find the two easiest points to identify the coordinates for.
Students who use a rule they do not understand or make mistakes when balancing equations.	Explain each step for balancing equations when modeling, including explaining how equality is maintained.	Ask students to explain how they solved and how they know that equality on both sides of the equation was maintained.

Title Slide

If students...

- get both sections correct:
 - start at You do
- miss the learning goal section only:
 - start at Let's Learn
- miss the prior learning section:
 - start at Prior Learning

Prior Learning

If stuck

- Remind students of the definitions of slope and y – intercept.
- Describe how to visualize the coordinates and the line. For example, from the point $(7, -1)$ we would go right on the x axis and down 4 on the y axis to get to $(9, -5)$.

Answers

$$\bullet \quad 1) \quad +2 \quad \left(\begin{array}{c} (7, 1) \\ (9, 5) \end{array} \right) \quad -4$$

$$\bullet \quad a) \quad m = \frac{\Delta y}{\Delta x} = \frac{-4}{2} = -2$$

$$\bullet \quad b) \quad \begin{aligned} -1 &= -2 \times 7 + b \\ -1 &= -14 + b \\ 13 &= b \end{aligned}$$

$$\text{Linear equation : } y = -2x + 13$$

$$\bullet \quad 2) \quad +2 \quad \left(\begin{array}{c} (-3, 7) \\ (-1, 10) \end{array} \right) \quad +3$$

$$\bullet \quad a) \quad m = \frac{\Delta y}{\Delta x} = \frac{3}{2} = 1.5$$

$$\bullet \quad b) \quad \begin{aligned} 10 &= 1.5 \times -1 + b \\ 10 &= -1 + b \\ 11.5 &= b \end{aligned}$$

$$\text{Linear equation : } y = 1.5x + 11.5$$

Let's learn

If stuck

- Model drawing a line and counting the data points above and below.
Note: It is also good to model drawing a line that doesn't work, discussing why and then redrawing.

Questions

- a) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- b) Do the points used to calculate the slope of the line have to be data points? (No, they just need to be points on the approximated line of best fit. It's best to choose points that will be easy to calculate. Always look for points on the grid lines.)
- b) What is the change in x , from 4.25 to 2.25? (-2 .)
- b) What is the change in y , from 25 to 40? ($+15$.)
- b) How does the slope connect to the graph? (The line decreases on the y axis by -7.5 for every $+1$ change in x .)

Questions: Second slide

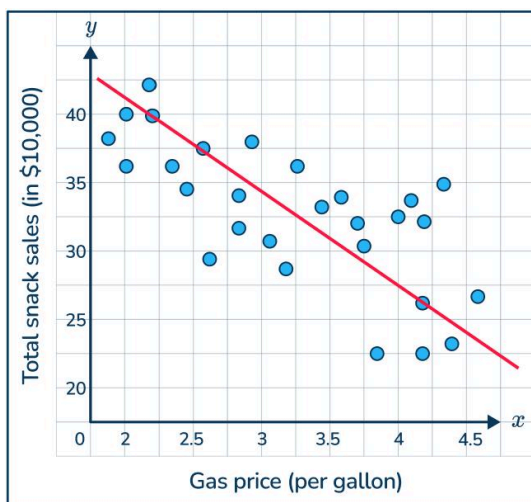
- a) What values can be substituted for x and y ? (Any set of coordinate from the line can be used.)
- b) How do you isolate b ? (Multiply the x value by -7.5 and then add the opposite of the product to both sides of the equation, leaving b isolated to one side.)
- b) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- b) How does the slope from the approximated line of best fit represent the data? (It shows that the total sales decrease by 7,500 for every \$1 increase in the gas price.)

Watch out for

- Students who have trouble identifying points on the line.
- Students who use a rule they do not understand to solve.
- Students who make mistakes balancing equations.

Answers

• a)



• b) $+2 \begin{pmatrix} (2.2, 40) \\ (4.2, 26) \end{pmatrix} -14$

• c) $m = \frac{\Delta y}{\Delta x} = \frac{-14}{2} = -7$

• d) $40 = -7 \times 2.2 + b$
 $40 = -15.4 + b$
 $55.4 = b$

• e) $y = -7x + 55.4$

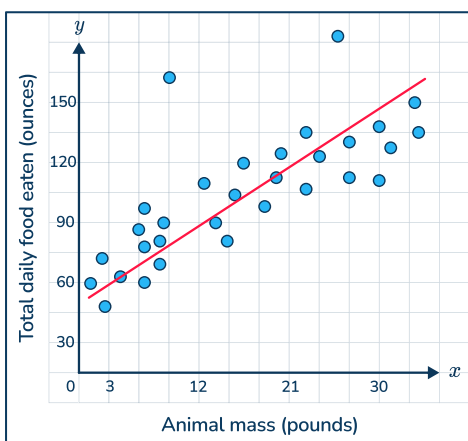
Follow me

Modeling prompts

- Draw a line through the middle of the data and explain why it is an approximate line of best fit.
- Identify two points from the line and use them to calculate the slope.
- Use the two points and the slope to find b .
- Give examples of data points that are close to the approximated line of best fit and data points that are not.

Answers

• a)



• b) $+16 \begin{pmatrix} (4, 63) \\ (20, 114) \end{pmatrix} +51$

• c) $m = \frac{\Delta y}{\Delta x} = \frac{51}{16} = 3.1875$

• d) $63 = 3.1875 \times 4 + b$
 $63 = 12.75 + b$
 $50.25 = b$

• e) $y = 3.1875x + 50.25$

• f) $(13.5, 90), (24, 130)$

• g) $(9, 162), (25.8, 184)$

Your turn

If stuck

- Remind students that the goal is to approximate the line of best fit.
- Prompt students to find two points on the line that have easy coordinates (aka ones that are on grid lines).
- Use similar guidance given in the Modeling prompts.

Questions

- a) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- b) Do the points used to calculate the slope of the line have to be data points? (No, they just need to be points on the approximated line of best fit. It's best to choose points that will be easy to calculate. Always look for points on the grid lines.)
- b) What is the change in x , from the first point to the second? (Answers will vary based on points students choose.)
- b) What is the change in y , from the first point to the second? (Answers will vary based on points students choose.)
- d) What values can be substituted for x and y ? (Any set of coordinates from the line can be used.)
- d) How do you isolate b ? (Multiply the x value by the slope and then subtract the product from both sides of the equation, leaving b isolated to one side.)
- e) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- e) How does the slope from the approximated line of best fit represent the data? (It shows that the daily waterpark visitors increase by 32 for every 1 degree increase in temperature.)

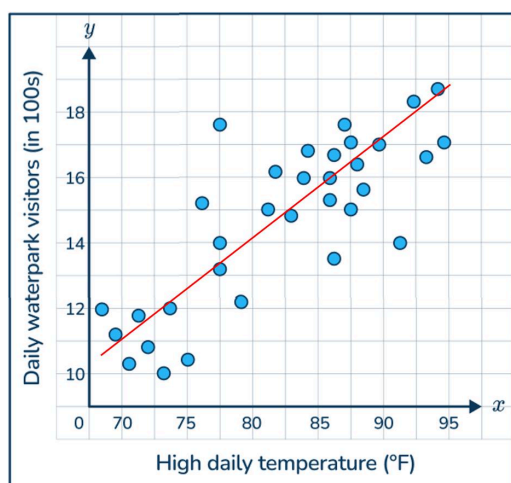
Watch out for

- Students who have trouble identifying points on the line.
- Students who use a rule they do not understand to solve.
- Students who make mistakes balancing equations.

Answers

- Note: Student answers may vary slightly if they choose different coordinates, but their approximated line of best fit should be close to the answer shown below.

• a)



• b) $+22.5 \left(\begin{array}{c} (70, 11) \\ (92.5, 18) \end{array} \right) +7$

• c) $m = \frac{\Delta y}{\Delta x} = \frac{7}{22.5} = 0.3\bar{1}$

• d) $11 = 0.3\bar{1} \times 70 + b$
 $11 = 21.\bar{7} + b$
 $-10.\bar{7} = b$

• e) $y = -10.\bar{7}x + 0.3\bar{1}$

- f) When the temperature is 70°F, there are 1,100 daily visitors to the waterpark.

When the temperature is 92.5°F, there are 1,800 daily visitors to the waterpark.

As the temperature increases, the number of daily visitors to the waterpark increases.

You do

If stuck

- Use the Support slide for question 3.

Questions: First slide

- a) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of best fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- a) Do the points used to calculate the slope of the line have to be data points? (No, they just need to be points on the approximated line of best fit. It's best to choose points that will be easy to calculate. Always look for points on the grid lines.)
- a) What is the change in x , from the first point to the second? (Answers will vary based on points students choose.)
- a) What is the change in y , from the first point to the second? (Answers will vary based on points students choose.)
- c) What values can be substituted for x and y ? (Any set of coordinates from the line can be used.)
- c) How do you isolate b ? (Multiply the x value by the slope and then subtract the product from both sides of the equation, leaving b isolated to one side.)
- d) How does the slope from the approximated line of best fit represent the data? (It shows that the gallons of water decrease by 15.5 for every 1 inch increase of rain each month.)

You do

Answers

- 1. a) $+8 \left(\begin{array}{c} (2, 245) \\ (10, 121) \end{array} \right) -124$
- b) $m = \frac{\Delta y}{\Delta x} = \frac{-124}{8} = -15.5$
- c) $121 = -15.5 \times 10 + b$
 $121 = -155 + b$
 $276 = b$
- d) Linear equation : $y = -15.5x + 276$
- e) $(2.8, 232), (10.8, 114)$
- f) $(8.8, 237), (4, 264)$

Questions: Second slide

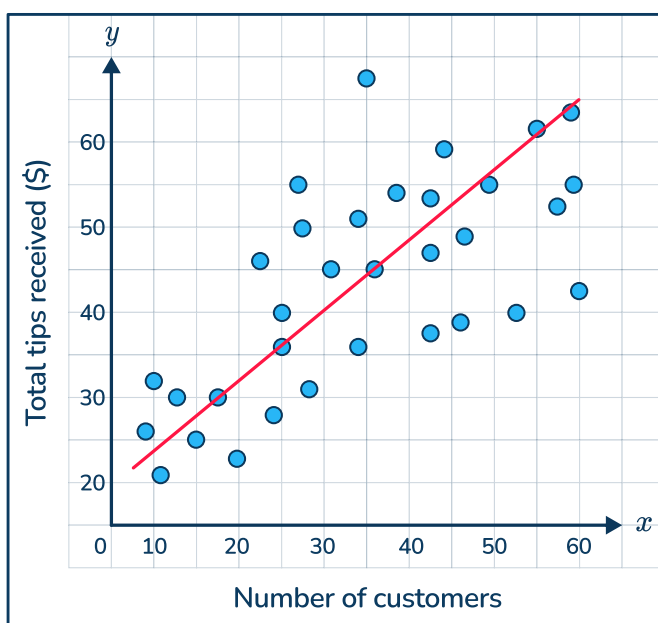
- a) Describe how you drew the approximated line of best fit. (Answers will vary.)
- b) What is the change in x , from the first point to the second? (Answers will vary based on points students choose.)
- b) What is the change in y , from the first point to the second? (Answers will vary based on points students choose.)
- b) What is a general form of linear equations? ($y = mx + b$.)
- b) What values can be substituted for x and y ? (Any set of coordinates from the line can be used.)
- b) How do you isolate
- b? (Multiply the x value by the slope and then subtract the product from both sides of the equation, leaving b isolated to one side.)
- b) How does the slope from the approximated line of best fit represent the data? (It shows that the total tips increase by 23 for each increase of 1 customer.)

Watch out for

- Students who have trouble identifying points on the line.
- Students who use a rule they do not understand to solve.
- Students who make mistakes balancing equations.

Answers

- 2. a)



Answers

• b) $+30 \begin{pmatrix} (30, 40) \\ (60, 65) \end{pmatrix} +25$

$$m = \frac{\Delta y}{\Delta x} = \frac{25}{30} = \frac{5}{6}$$

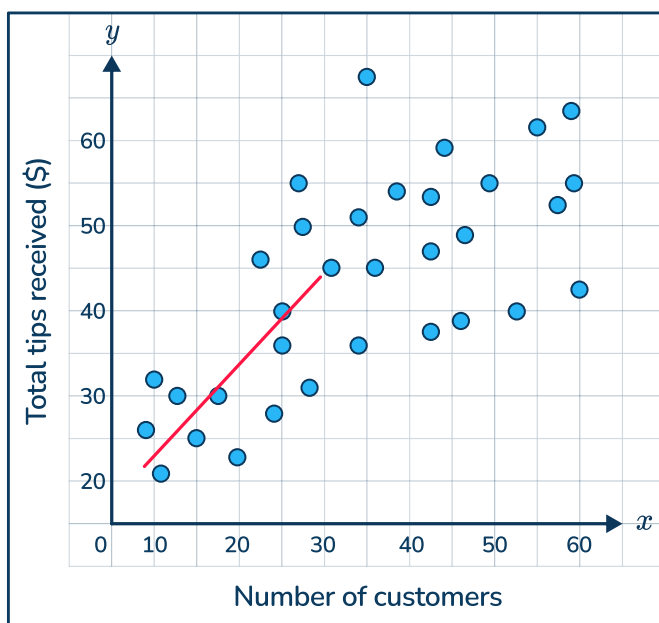
$$40 = \frac{5}{6} \times 30 + b$$

$$40 = 25 + b$$

$$15 = b$$

$$y = \frac{5}{6}x + 15$$

• 3. a)



• b) $-20 \begin{pmatrix} (30, 44) \\ (10, 23) \end{pmatrix} -21$

$$m = \frac{\Delta y}{\Delta x} = \frac{-21}{-20} = 1 \frac{1}{20} = 1.05$$

$$44 = 1.05 \times 30 + b$$

$$44 = 31.5 + b$$

$$12.5 = b$$

$$y = 1.05x + 12.5$$

Go further

If stuck

- Remind students that the goal is to approximate the line of best fit and help them draw the line.
- Prompt students to find two points on the line that have easy coordinates (aka ones that are on grid lines).

Questions: First slide

- How did the change in scale change how the data was displayed? (It is the same data points, but they look farther apart, since the scale is smaller. The data points still have the same overall pattern - since it is the same data - but overall they are more spread out.)
- How does the equation of this line compare to your equation from question 3 (previous slide)? (Answers will vary.)

Note: Equation may be exact, but may vary, since the change in scale may lead students to identify slightly different points.

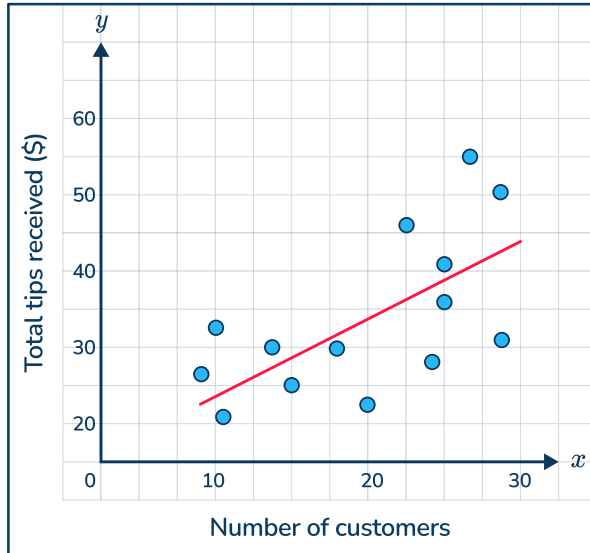
Watch out for

- Students who do not realize that a changed scale does not change the data - just the way it is visualized.

Go further

Answers

• a)



• b) $-20 \begin{pmatrix} (30, 44) \\ (10, 23) \end{pmatrix} -21$

$$m = \frac{\Delta y}{\Delta x} = \frac{-21}{-20} = 1\frac{1}{20} = 1.05$$

• c) $44 = 1.05 \times 30 + b$

$$44 = 31.5 + b$$

$$12.5 = b$$

Linear equation : $y = 1.05x + 12.5$

Support for Slide(s)

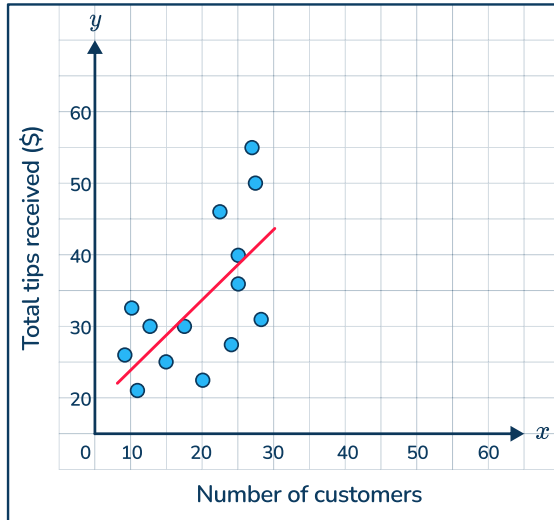
Questions

- a) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- b) Do the points used to calculate the slope of the line have to be data points? (No, they just need to be points on the approximated line of best fit. It's best to choose points that will be easy to calculate. Always look for points on the grid lines.)
- b) What is the change in x , from the first point to the second? (Answers will vary based on points students choose.)
- b) What is the change in y , from the first point to the second? (Answers will vary based on points students choose.)
- c) What values can be substituted for x and y ? (Any set of coordinates from the line can be used.)
- c) How do you isolate b ? (Multiply the x value by the slope and then subtract the product from both sides of the equation, leaving b isolated to one side.)
- c) Why is this an approximate line of best fit? (The line was drawn through the middle of the data as an estimate, but an exact line of fit calculates the distance of the line from each point to minimize it, which is usually done by a modeling software.)
- c) How does the slope from the approximated line of best fit represent the data? (It shows that the total tips increase by 123 for each increase of 1 customer.)
- c) Why is the slope for 0–30 customers larger than the slope in question 2? (This part of the line is steeper, so by just looking at these data points, the approximated line of best fit would have a greater slope.)

Support for Slide(s)

Answers

• a)



• b) $-20 \begin{pmatrix} (30, 44) \\ (10, 23) \end{pmatrix} -21$

$$m = \frac{\Delta y}{\Delta x} = \frac{-21}{-20} = 1\frac{1}{20} = 1.05$$

• c) $44 = 1.05 \times 30 + b$

$$44 = 31.5 + b$$

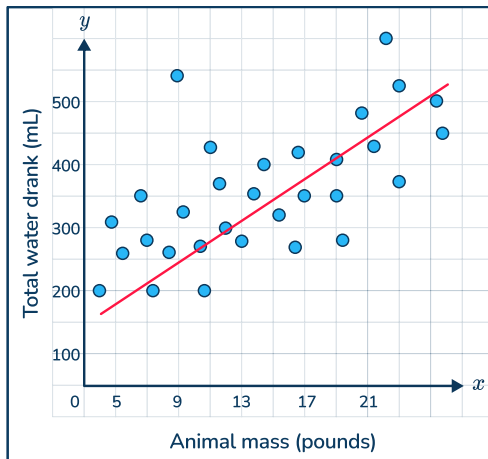
$$12.5 = b$$

Linear equation : $y = 1.05x + 12.5$

Assessment question:

Correct answer:

$$y = 14.3x + 149.9$$

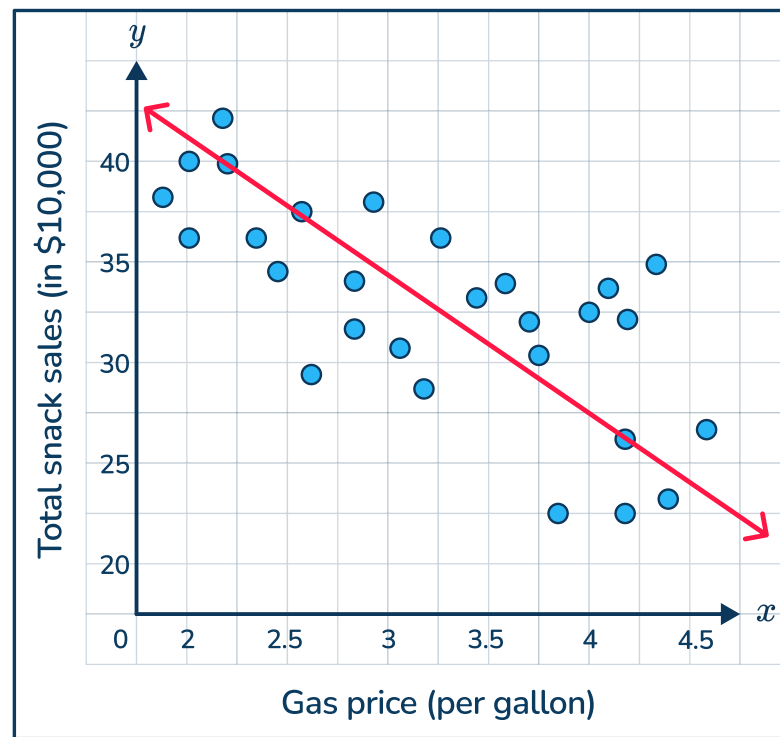


Today you will learn about

Informally fitting bivariate data with a line of best fit



Learning Goal



Write the equation for the approximated line of best fit.

.....

Prior Learning

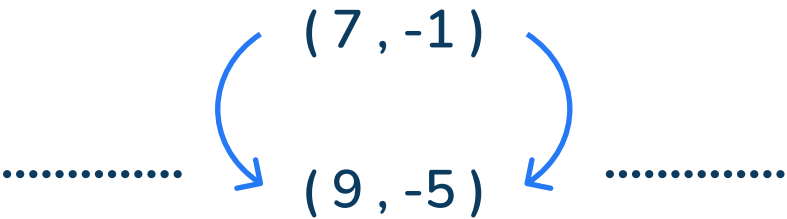
What is the equation of a linear function that passes through (7,-1) and (9,-5)?

.....

Prior learning

Before we can informally fit bivariate data with a line of best fit, we need to know how to **construct an equation to model a linear function using coordinates.**

1



a

Identify the rate of change. $m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\dots\dots\dots}}{\boxed{\dots\dots\dots}} = \dots\dots\dots$

b

Find b by substituting values into $y = mx + b$.

$\dots\dots\dots = \dots\dots\dots \times \dots\dots\dots + b$

$\dots\dots\dots = \dots\dots\dots + b$

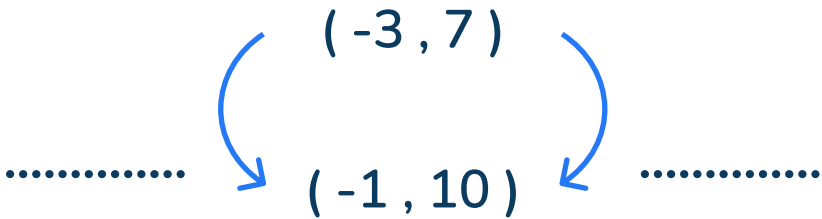
$\dots\dots\dots = b$

For x and y use one of the coordinates given.

Linear equation:

 $\dots\dots\dots$

2



a

Identify the rate of change. $m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\dots\dots\dots}}{\boxed{\dots\dots\dots}} = \dots\dots\dots$

b

Find b by substituting values into $y = mx + b$.

$\dots\dots\dots = \dots\dots\dots \times \dots\dots\dots + b$

$\dots\dots\dots = \dots\dots\dots + b$

$\dots\dots\dots = b$

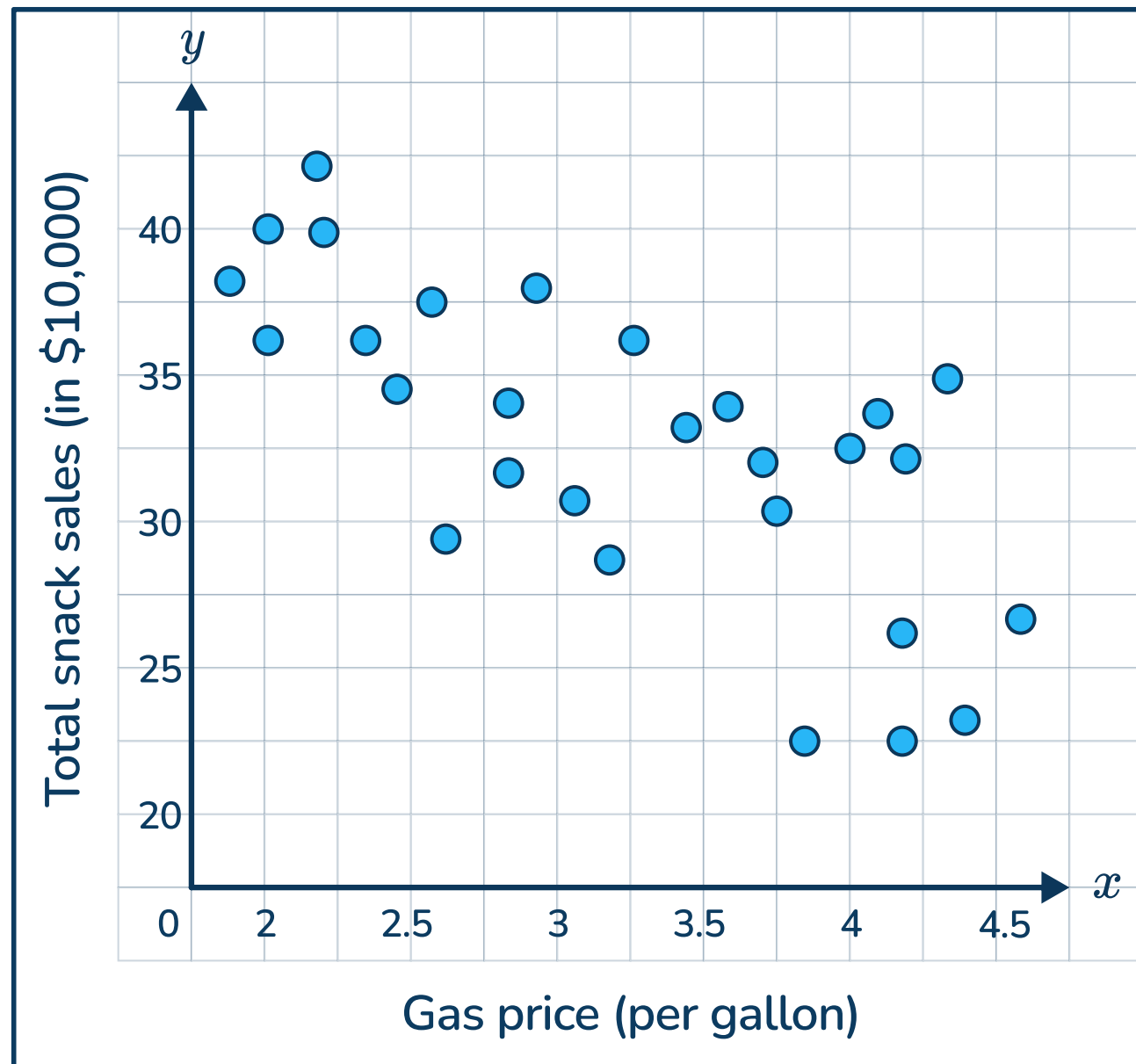
Linear equation:

 $\dots\dots\dots$

Let's learn

Bivariate data that has a linear association can be defined by a **line of best fit**.

We can use what we know about linear relationships to approximate a line of best fit.



a Draw a line that goes through the middle of the data. It should have about the same amount of data points above and below.

b Identify the ordered pairs of two points on the line.

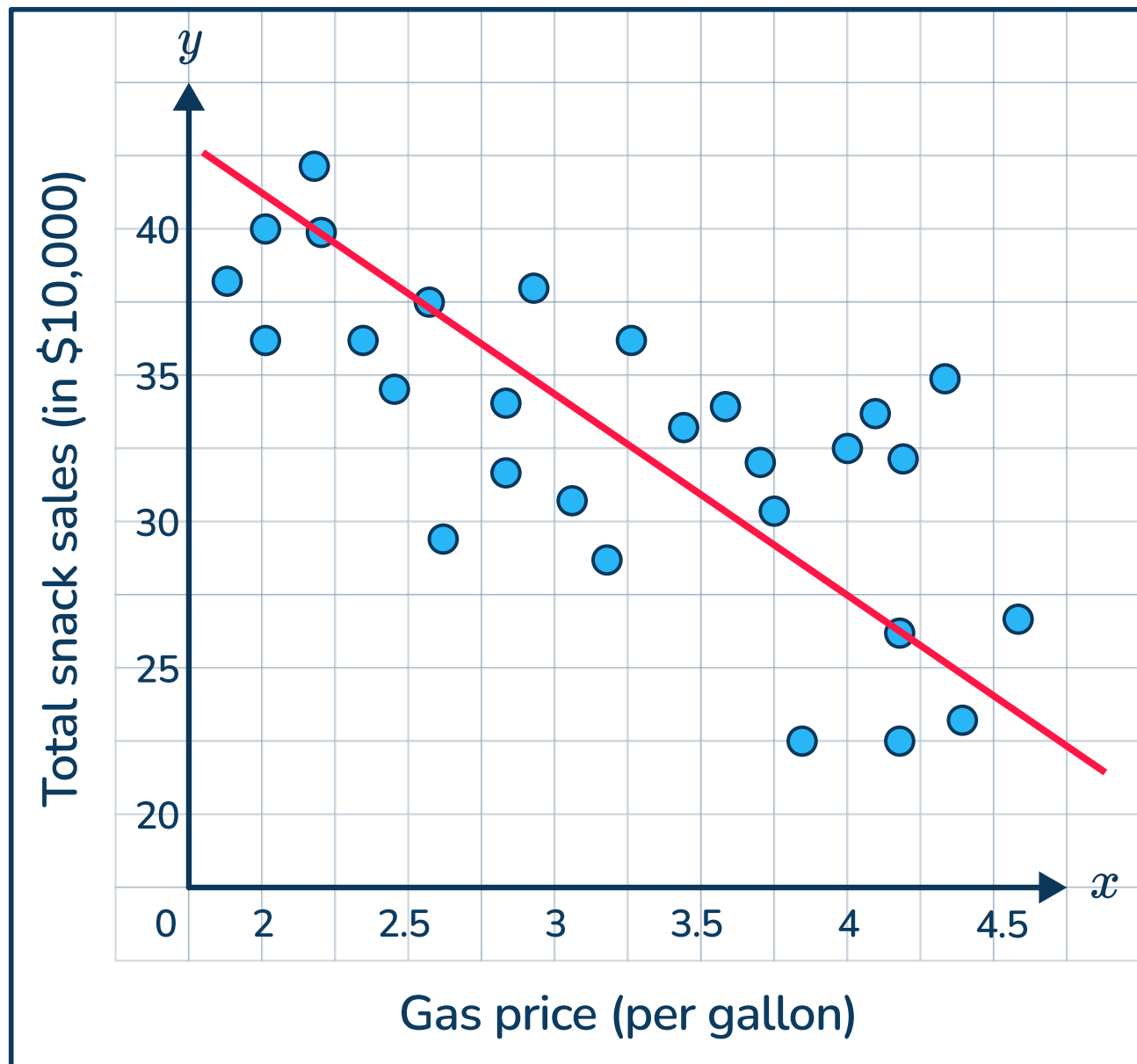
..... $\left(\begin{array}{c} \text{.....} \\ \text{.....} \end{array} , \begin{array}{c} \text{.....} \\ \text{.....} \end{array} \right)$

c Identify the rate of change. $m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\text{.....}}}{\boxed{\text{.....}}} = \text{.....}$

Let's learn

Bivariate data that has a linear association can be defined by a **line of best fit**.

Once we identify the rate of change, we represent the approximated linear relationship in the form $y = mx + b$



If the scale of the x -axis does not start at 0, we cannot use the graph to find the y -intercept.

d Find b by substituting values into $y = mx + b$

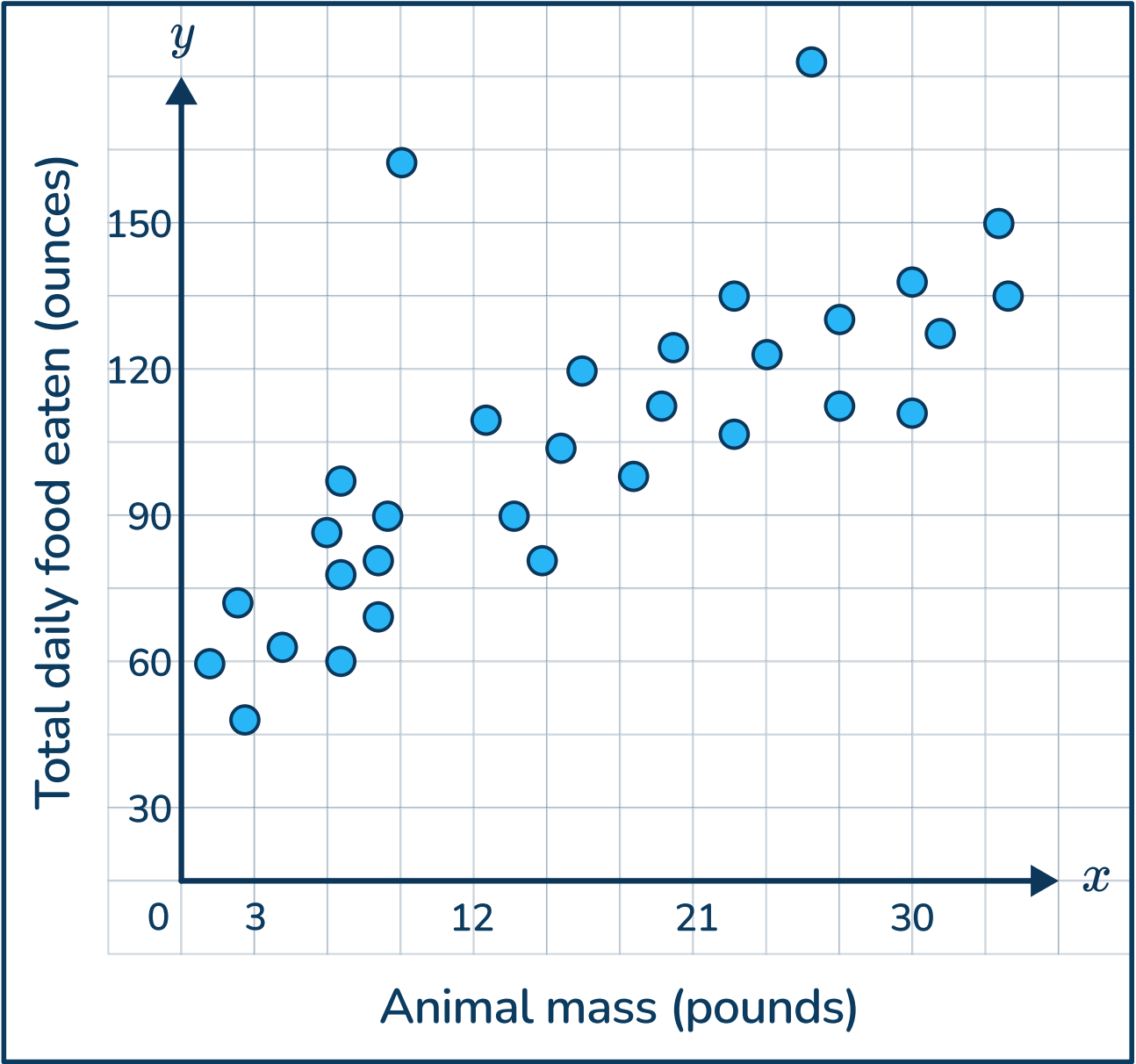
$$\text{.....} = \text{.....} \times \text{.....} + b$$

$$\text{.....} = \text{.....} + b$$

$$\text{.....} = b$$

e Linear equation:

Follow me



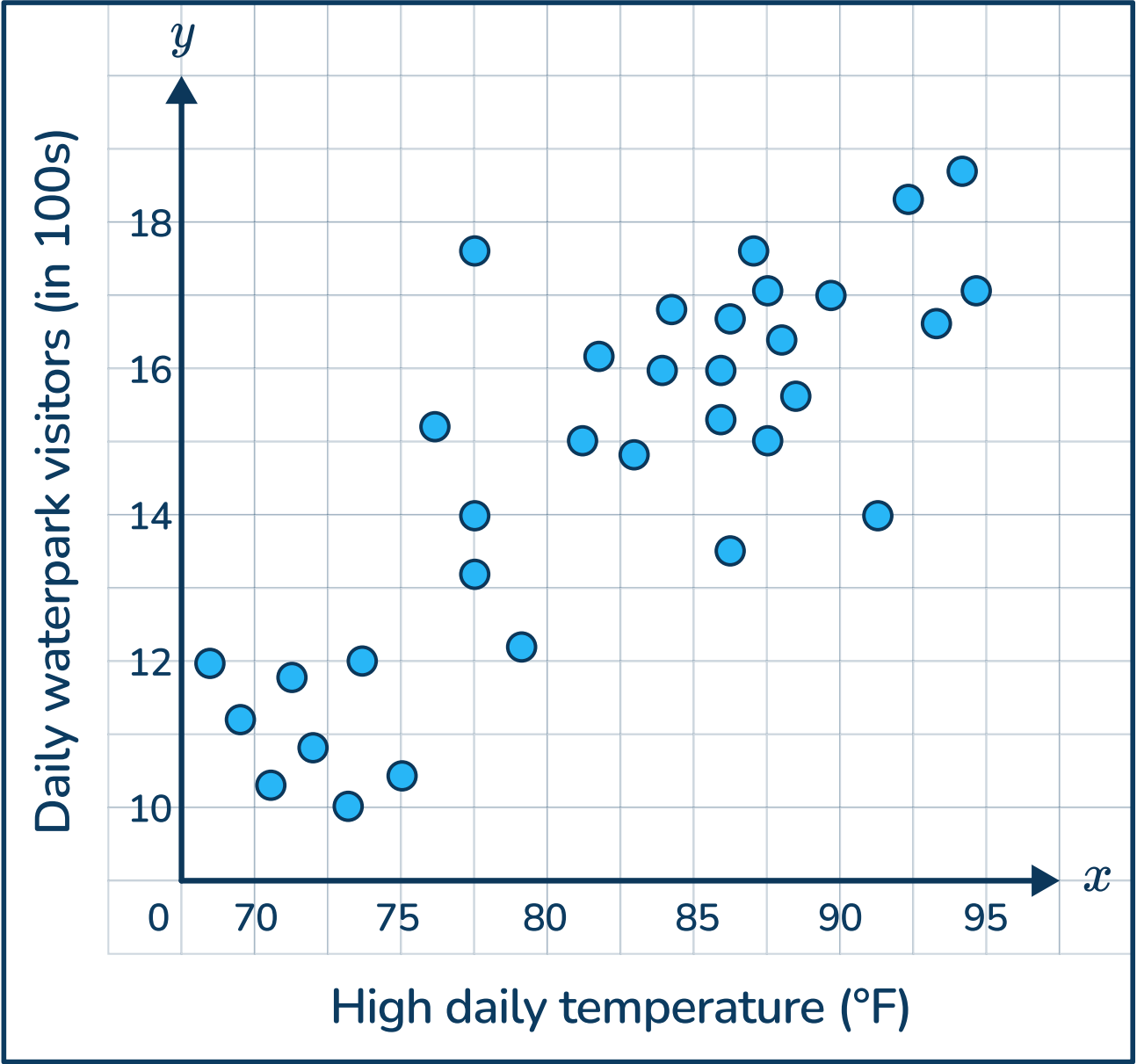
- a Draw an approximate line of best fit.
- Always consider how outliers will affect the line of best fit.

- b Identify the ordered pairs of two points on the line.

(..... ,)
(..... ,)
- c Identify the rate of change.
$$m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\text{.....}}}{\boxed{\text{.....}}} = \text{.....}$$
- d Find b by substituting values into $y = mx + b$.
$$\text{.....} = \text{.....} \times \text{.....} + b$$
$$\text{.....} = \text{.....} + b$$
$$\text{.....} = b$$

- e Linear equation:
.....
- A line's fit describes how close the data points are to the line of best fit.
- f Identify data points that are close to the line of best fit.
(..... ,)
- g Identify data points that are NOT close to the line of best fit.
(..... ,)

Your turn



a Draw an approximate line of best fit.

b Identify the ordered pairs of two points on the line.

..... (..... ,)
..... (..... ,)

c Identify the rate of change.

$$m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\text{.....}}}{\boxed{\text{.....}}} = \text{.....}$$

d Find b by substituting values into $y = mx + b$.

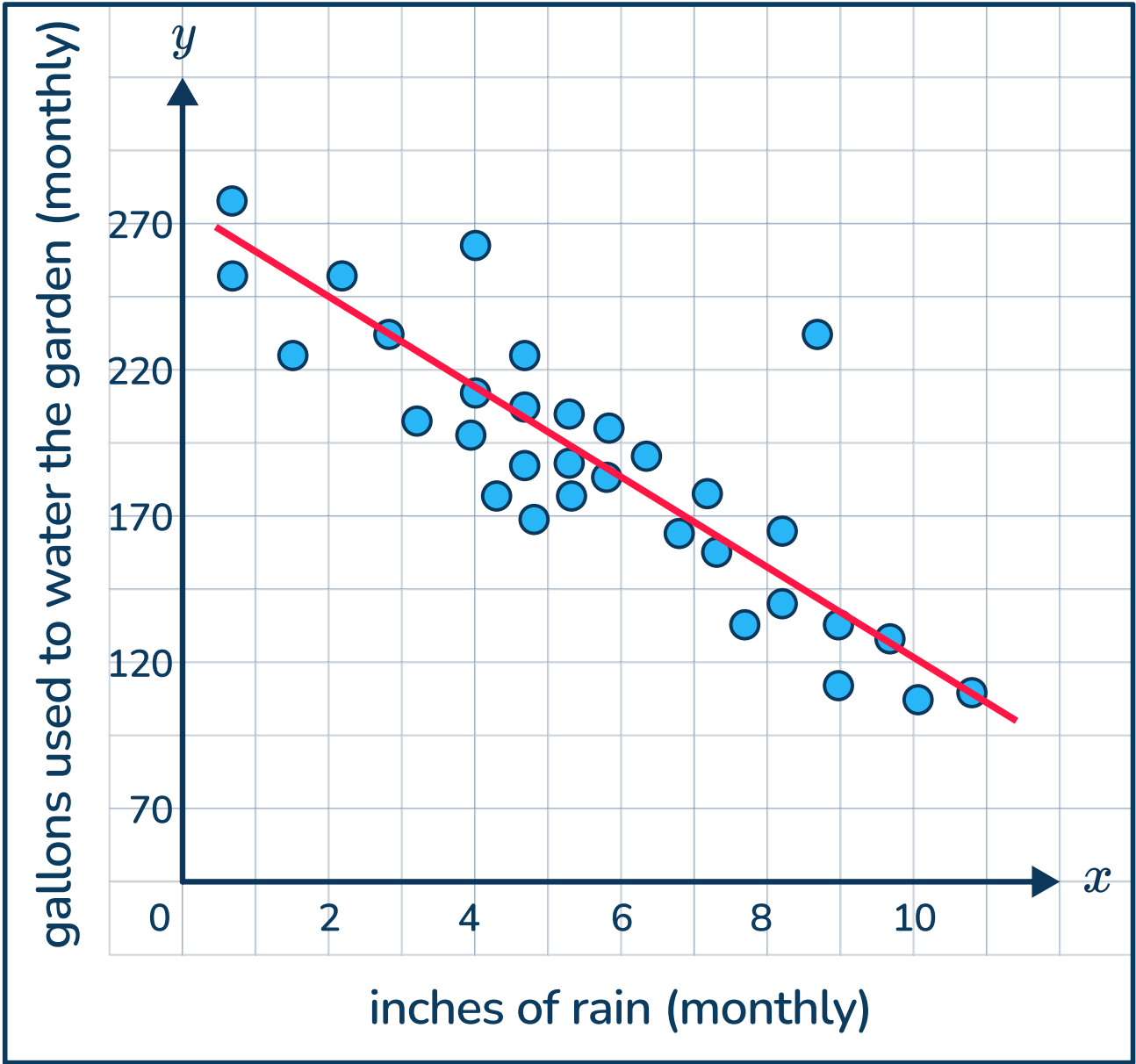
..... = \times + b
..... = + b
..... = b

e Linear equation:

.....

f Describe the line's fit, including using at least two specific ordered pairs.

1



a Identify the ordered pairs of two points on the line.



b Identify the rate of change. $m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\text{.....}}}{\boxed{\text{.....}}} = \text{.....}$

c Find b by substituting values into $y = mx + b$.

..... = \times + b

..... = + b

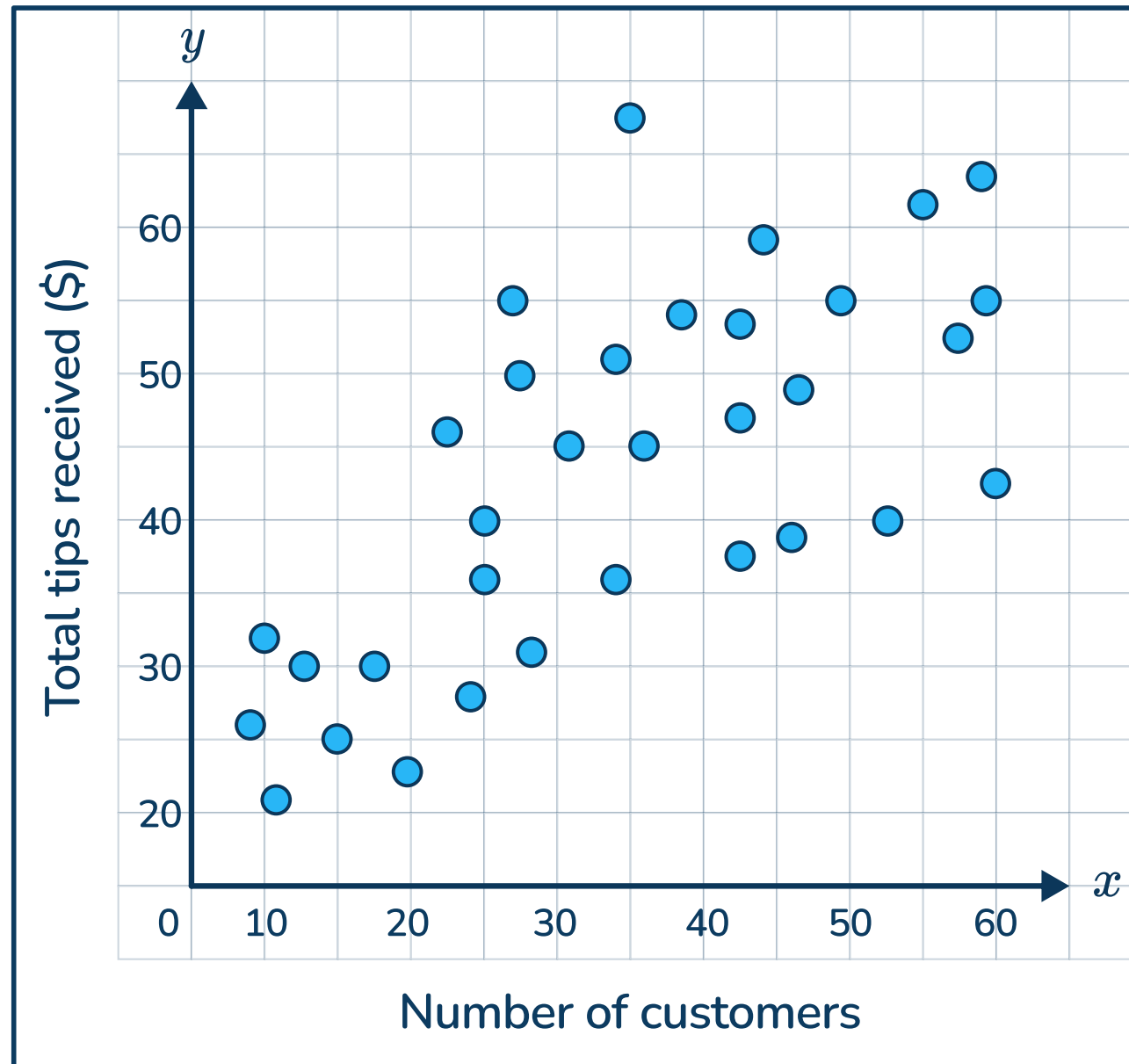
..... = b

d Linear equation:
.....

e Identify data points that are close to the line of best fit.
(..... ,)

f Identify data points that are NOT close to the line of best fit.
(..... ,)

2



a Draw an approximate line of best fit.

.....

b Write the equation for the line of best fit.

3

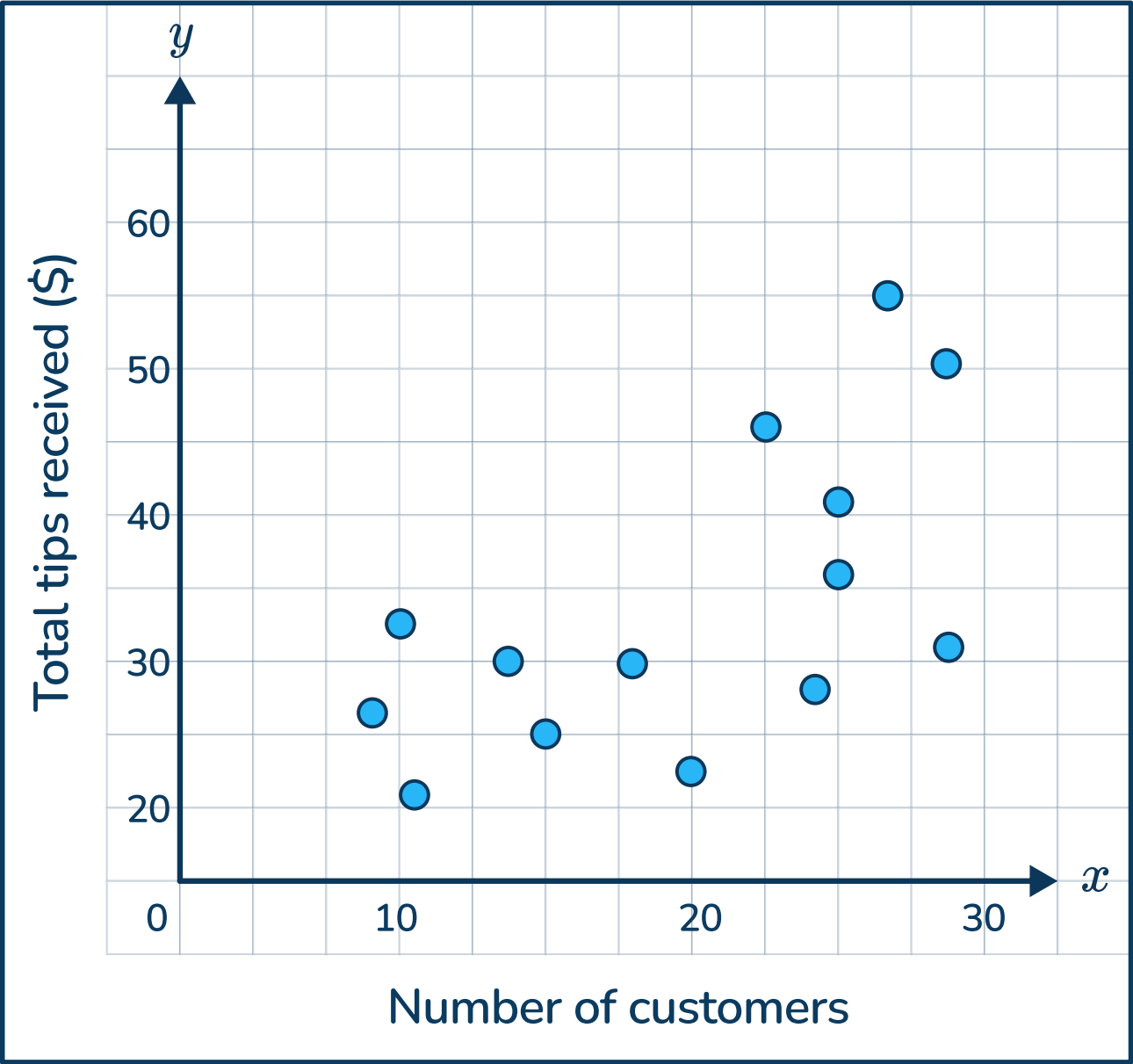
a Draw an approximate a line of best fit for 0-30 customers.

b Write the equation for the line of best fit for 0-30 customers.

.....

Go further

Let's look at the same data in a graph with a different scale.



- a Draw an approximate line of best fit.
- b Write the equation for the line of best fit.

.....



Let's look at this one more closely!



a Draw an approximate line of best fit.

b Identify the rate of change. $m = \frac{\Delta y}{\Delta x} = \frac{\boxed{\dots\dots\dots}}{\boxed{\dots\dots\dots}} = \dots\dots\dots$

c Find b by substituting values into $y = mx + b$.

$\dots\dots\dots = \dots\dots\dots \times \dots\dots\dots + b$

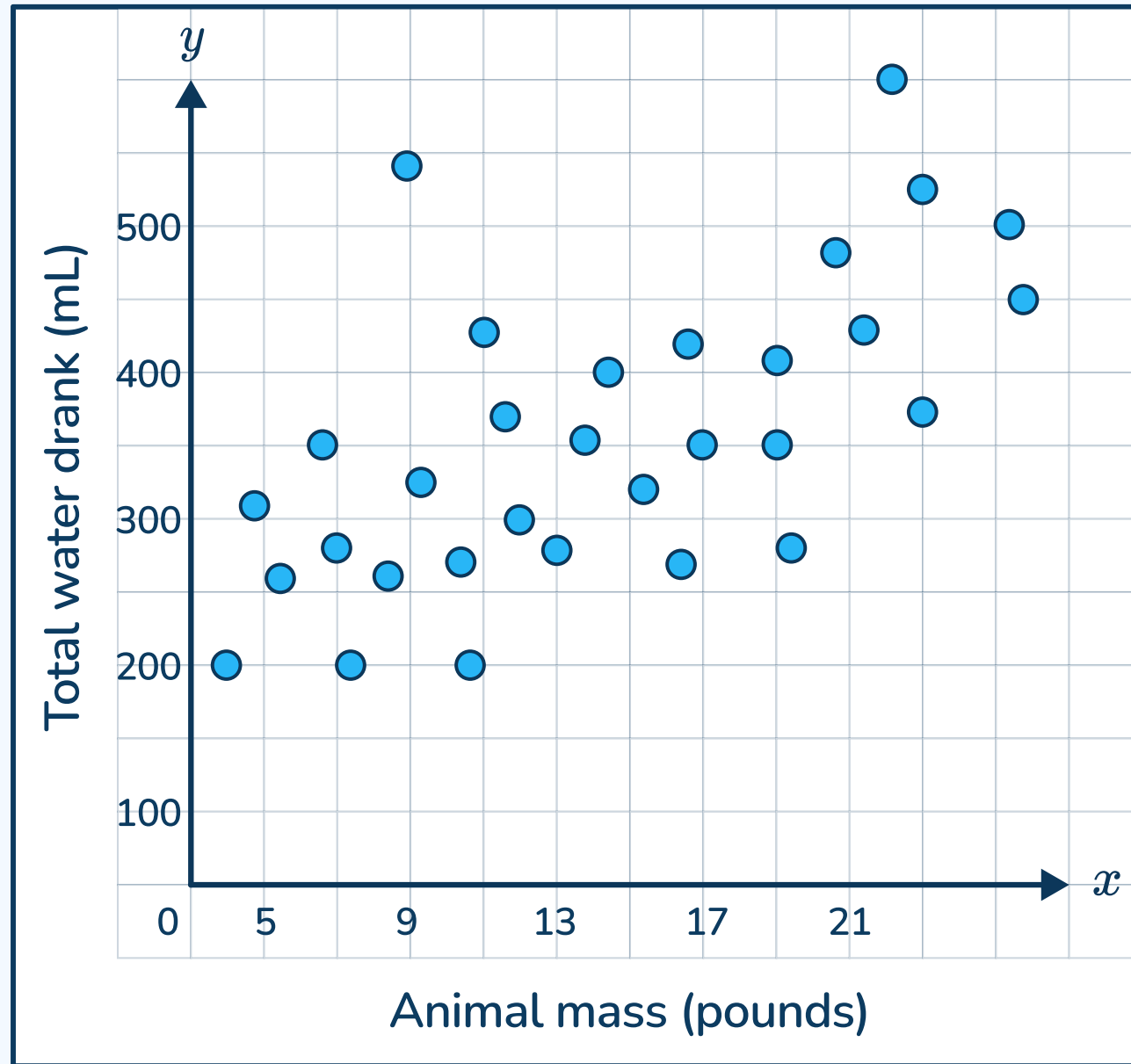
$\dots\dots\dots = \dots\dots\dots + b$

$\dots\dots\dots = b$

Linear equation:

$\dots\dots\dots$

Check your understanding



Write the equation for an approximated line of best fit.

.....

Why do I need to try this question on my own first?

- To show your tutor what you understand
- To give you more practice
- To show your teacher how you are doing



Do you have a group of students who need a boost in math?

Each student could receive personalized lessons every week from our specialist one-on-one math tutors.




- ✓ Differentiated instruction for each student
- ✓ Aligned to your state's standards
- ✓ Scaffolded learning to close gaps

“We just had our first session and it went great! The kids really liked it and felt like they were learning! One even said he finally felt like math was making sense.”



Michelle Craig, Instructional Coach,
Sherwood Forest Elementary, Washington

Speak to us

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