



**THIRD SPACE
LEARNING**

Math Intervention Pack

Identifying rational and
irrational numbers

Grade 8

How To Use This Resource

1. Title Slide

Use this slide to activate prior knowledge needed for lesson. Students should be encouraged to initially attempt the question presented independently.

2. Prior Learning

Use this slide to review the knowledge that will be required to be successful in this lesson. If students feel confident on the prior learning section of the Title Slide then this slide can be skipped

3. Let's Learn

Use this slide to introduce the concept. Tutors should work with the student to explore the concept together, usually using diagrams to support understanding.

4. Follow Me + Your Turn

The tutor should work through the follow me slide, modeling the process and explaining their thinking out loud.

Students should use the your turn slide as an opportunity to work through a question similar to the follow me questions. They should apply the method modeled by the tutor in the follow me slide. Students should be encouraged to explain their thinking out loud.

5. You Do

Students should work through a range of questions that build in complexity.

Tutors can offer support but students should initially be encouraged to attempt these questions independently.

6. Go Further

Use this slide to allow students to apply their understanding to a more challenging question in an unfamiliar context.

How To Use This Resource

7. Support for Slides

The support slide is used to support students during the lesson. In the tutor notes, there will be guidance as to when to use the support slide.

8. Check Your Understanding

Tutors should use this slide to assess the student's knowledge and whether or not they have mastered the concept within the lesson.

Standard

8.NS.A.1 - Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Key Mathematical Ideas

1. Apply understanding of and rational numbers to classify numbers.
2. Given a number, identify it as being rational or irrational.

Overview

Terminology

- **Repeating decimal** - A decimal number that contains a digit or group of digits that repeats endlessly.
- **Rational number** - A rational number is a number that can be written as a fraction where the numerator and denominator are both integers, and the denominator is not zero.
- **Irrational number** - An irrational number is a number that cannot be written as the ratio of two integers. In other words, it cannot be written as a fraction where the denominator is not equal to 0.

Sentence Stems

- _____ numbers cannot be written as _____.
- _____ numbers can be written as _____.
- Repeating decimals are _____ numbers because they can be written as _____.
- Square roots of non-perfect square numbers are _____ numbers because they cannot be written as _____.

Overview

Common Misconceptions

Common Misconceptions	Tutoring Strategies	Checks for Understanding
Students who do not recognize perfect square numbers.	Before discussing the classification of the square of non-perfect square numbers, review the meaning of a perfect square number.	When students are classifying numbers such as $\sqrt{5}$ or $\sqrt{4}$ on the “Your turn” or “You do” slide have them provide an explanation to you.
Students that struggle with identifying repeating decimals vs non-repeating decimals.	Point out numbers such as, 2.353353335....as non-repeating and why it is non-repeating.	On the “you do” slide have students explain their thinking to you when they classify the numbers.

Title Slide

If students...

- get both sections correct:
 - start at You do
- miss the learning goal section only:
 - start at Let's Learn
- miss the prior learning section:
 - start at Prior Learning

Prior learning

If stuck

- Do more examples to help students recall the sets of numbers.
- Reinforce that natural numbers are also referred to as “counting number.”
- Emphasize that 0 is part of the whole number system but not the natural number system.

Let's learn

If stuck

- Provide students with more examples of irrational numbers so they can make connections and see patterns.
- Have students use an online calculator.

Questions

- a) What do you notice about the number π ? (It goes on endlessly. There is no repetition to the digits.)
- a) Using your knowledge of changing decimal numbers to fractions, do you think π can be written as a fraction? (Pause to have students apply their knowledge to see if they can change it to a fraction before answering. No, it cannot be written as a fraction because the decimal is endless and does not repeat.)
- a) Do you think repeating decimals can be written as fractions? (Yes, in a prior lesson we learned how to convert repeating decimals to fractions. Also look to the support slide for assistance.)
- b) What do you notice about the number $\sqrt{2}$? (Similar to π the decimal is endless and does not repeat.)
- b) Is 2 a perfect square? (Have students explain what they think a perfect square number is before they answer. No, 2 is not a perfect square number.)
- b) Using your knowledge of changing decimal numbers to fractions, do you think that $\sqrt{2}$ can be written as a fraction? (Pause to have students apply their knowledge to see if they can change it to a fraction. No, it cannot be written as a fraction.)
- c) What do you notice about the $\sqrt{5}$? (Like the other examples, it is a decimal that is endless and does not repeat.)
- c) Is 5 a perfect square number? (No, it's not a perfect square number.)
- c) Do you think the square root of non-perfect square numbers can be integer answers? (No, the square root of non-perfect square numbers cannot be integer or fraction answers.)

- c) Do you think $\sqrt{5}$ can be written as a fraction? (Pause to have students apply their knowledge to see if they can change it to a fraction before they answer. No, I do not think $\sqrt{5}$ can be written as a fraction.)
- d) What did you notice about the $\pi, \sqrt{2}, \sqrt{5}$? (I noticed that they were all decimal numbers that didn't end (endless), no digit repeated in the decimal numbers, and they were not able to be written as fractions.)
- d) What are these numbers called? (They are irrational numbers.)

Watch out for

- Students who have difficulty identifying perfect square numbers vs non-perfect square numbers.

Answers

- a) No. π is an irrational number so it cannot be expressed as a fraction.
- b) No. $\sqrt{2}$ cannot be expressed as one integer, divided by another.
- c) No. $\sqrt{5}$ is an irrational number.
- d) $\pi, \sqrt{2}, \sqrt{5}$ are all irrational numbers as they cannot be written as one integer, divided by another.

Follow me

Modeling prompts

- Point out that part a is not a repeating decimal.
- Explain that even numbers are not necessarily perfect squares.

Answers

- 2.353353335... is irrational
- $1.\bar{3}$ is rational ($=1\frac{1}{3}$)
- $\sqrt{9}$ is rational ($=3$)
- $\sqrt{18}$ is irrational

Your turn

If stuck

- Use similar guidance given in the Modeling prompts.

Questions

- a) Is $\sqrt{25}$ rational or irrational? (It is rational.)
- a) Why $\sqrt{25}$ is rational? (Because 25 is a perfect square number, $\sqrt{25} = 5$.)
- b) Is 0.747747774... rational or irrational? (It's irrational.)
- b) Why is 0.747747774 irrational? (The number is irrational because it is endless (non-terminating) and does not have any repeating digit so it can't be written as a fraction.)
- c) Is 0.004 rational or irrational? (0.004 is rational)
- c) Why is 0.004 rational? (It is rational because it can be written as a fraction $\frac{4}{1000}$ and it is a decimal that ends (terminates).)
- d) Is $\sqrt{12}$ rational or irrational? (It is irrational.)
- d) Why is $\sqrt{12}$ irrational? ($\sqrt{12}$ is irrational because 12 is not a perfect square number. $\sqrt{12}$ also cannot be written as a fraction.)

Watch out for

- Students who have difficulty identifying repeating vs non-repeating decimals like the number in part b.
- Students that think that all even numbers are perfect squares.

Answers

- $\sqrt{25}$ is rational (=5)
- 0.747747774... is irrational
- 0.004 is rational ($= \frac{4}{1000}$)
- $\sqrt{12}$ is irrational

You do

If stuck

- Use the Support slide changing 0.222...and 0.41414...into a fraction.

Questions

- Which numbers should be placed under the rational column? (0.25, $\sqrt{100}$, 3.414141..., 5.5, $\sqrt{1}$, $\sqrt{\frac{1}{4}}$, 1.009, $0.\bar{2}$)
- Why did you place each of those numbers in the Rational column? (Because each of those numbers could be written as a fraction or whole number.)

$$\sqrt{100} = 10$$

$$3.414141... = 3.\frac{41}{99} \text{ repeating decimals are rational}$$

$$5.5 = 5\frac{1}{2} = \frac{11}{2}$$

$$\sqrt{1} = 1$$

$$\sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$1.009 = 1\frac{9}{1000}$$

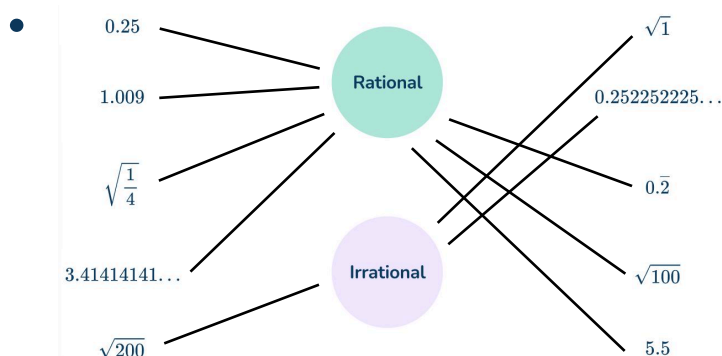
$$0.\bar{2} = \frac{2}{9} \text{ repeating decimals are rational}$$

- Which numbers should be in the Irrational column? (0.252252225..., $\sqrt{200}$.)
- Why did you place each of those numbers in the Irrational column? (Because those numbers cannot be written as fractions.)

Watch out for

- Students that identify decimals such as 0.252252225...as repeating decimals.
- Students that think all even numbers are perfect square numbers.

Answers



Go further

If stuck

- Remind students the difference between square root vs cube root.

Questions

- a) How can you change the repeating decimal to a fraction? (Have students explain the process.)
- a) What is the correct classification of this number? (Repeating decimals are rational. The fraction equivalent is $\frac{5}{9}$.)
- b) Can you change this decimal to a fraction? (No, it cannot be changed to a fraction.)
- b) What is the correct classification? (Irrational)
- c) Is 8 a perfect cube? (Yes, 8 is a perfect cube.)
- c) What is the $\sqrt[3]{8}$? ($\sqrt[3]{8} = 2$, because $2 \times 2 \times 2 = 8$.)
- c) What is the correct classification for $\sqrt[3]{8}$? (Rational)
- d) Can the number be written as a fraction? (Have the student explain the process of how to write it as a fraction.)
- d) What is the correct classification of this number? (Repeating decimals are rational and the fraction equivalent is $\frac{74}{99}$.)

Watch out for

- Students who have difficulty identifying repeating decimals vs non-repeating decimals.

Answers

- 0.555555... rational ($= \frac{5}{9}$)
- 0.616616661... irrational
- $\sqrt[3]{8}$ rational ($= 2$)
- 0.7474747474... rational ($= \frac{74}{99}$)

Support for Slide(s)

You Do:

Questions

- When converting 0.54 to a fraction, why is the initial denominator 100? (It's 100 because the 0.54 last digit is in the hundredths place which becomes the denominator.)
- When converting 0.54 to a decimal, what is the numerator when the denominator is 100? (The numerator should be 54.)
- What does $\frac{54}{100}$ simplified to be? (It simplifies to $\frac{27}{50}$.)
- What is the denominator of the fraction represented by 0.43? (The denominator is 1000 because 3 is the last digit of the decimal and is in the thousandths place.)
- How should the fraction be written? (0.043 as a fraction is $\frac{43}{1000}$.)
- Can the fraction be simplified? (No, the fraction cannot be simplified.)
- As you go through the steps of converting a repeating decimal to a fraction, ask students why in Step 2 the equation is multiplied by 10? (The equation is multiplied by 10 because only one digit is repeated.)
- Ask students what would they multiply by if two or three digits were repeating? (Two digit repeating multiply by 100, three digit repeating multiply by 1000.)
- Provide them with another example if they struggle.

Answers

- $0.54 = \frac{54}{100} = \frac{27}{50}$
- $0.043 = \frac{43}{1000}$
- $x = \frac{7}{9}$

Check your Understanding

Correct answer:

- a) Irrational (non-perfect square)
- b) Rational (repeating decimal)
- c) Irrational (non-repeating decimal)

Today you will learn about

Identifying rational and irrational numbers



Learning Goal

Identify the numbers as either rational or irrational.

a $\sqrt{4}$

.....

b $0.121121112\dots$

.....

c $\sqrt{10}$

.....

d $0.3333333\dots$

.....

Prior Learning

Identify the numbers as either an integer or a rational number.

a -9

.....

b 1200

.....

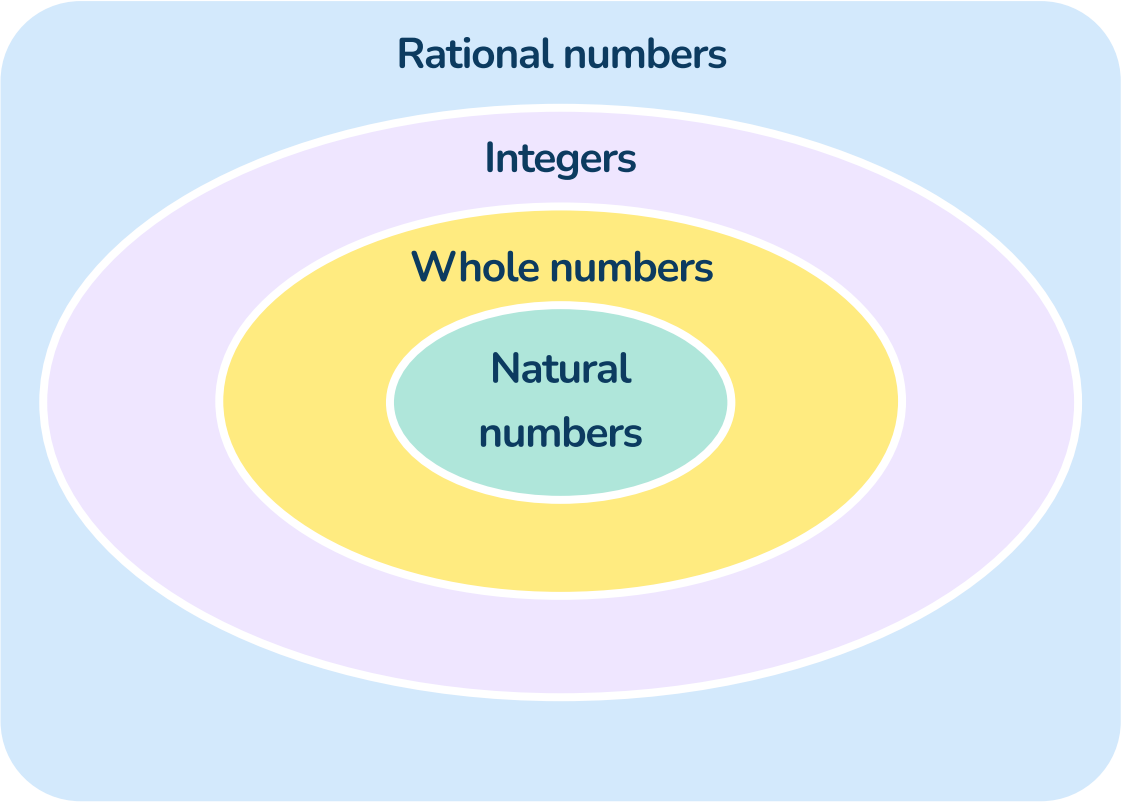
c $-\frac{2}{7}$

.....

Prior learning

Before we can identify rational and irrational numbers, we first must understand the sets of numbers.

Let's take a look at this Venn diagram which shows different sets of numbers.



a Using the diagram, is the statement true:
Integers are also rational numbers.

.....

b Using the diagram, is the statement true:
Fractions are also natural numbers.

.....

c Give an example of a natural number.

.....

d Give an example of a whole number
that is not a natural number.

.....

e Give an example of an integer that is
not a whole number.

.....

Any number that can be written as a fraction is rational.

Let's learn

Numbers that cannot be classified as **rational numbers** are called **irrational numbers**.

Numbers that cannot be written as the ratio of two non-zero integers are irrational.

Let's look at some examples

a $\pi = 3.1415926535 \dots$

Do you think π can be written as a fraction?

b $\sqrt{2} = 1.41421356237 \dots$

Do you think $\sqrt{2}$ can be written as a fraction?

c $\sqrt{5} = 2.236067977 \dots$

Do you think $\sqrt{5}$ can be written as a fraction?

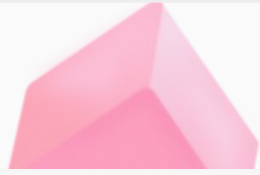
d What do the numbers π , $\sqrt{2}$, $\sqrt{5}$ have in common?
.....

Non-repeating, non-terminating decimals cannot be written as fractions.

Square roots of non-perfect square numbers are irrational.



Follow me



Identify the numbers as rational or irrational.

a $2.353353335\dots$

b $1.\overline{3}$

c $\sqrt{9}$

d $\sqrt{18}$

Repeating decimals are rational.



Your turn



Identify the numbers as rational or irrational.

a $\sqrt{25}$

b $0.747747774\dots$

c 0.004

d $\sqrt{12}$

You do

Draw an arrow to match the number to the right set.

0.25

1.009

$\sqrt{\frac{1}{4}}$

3.41414141...

$\sqrt{200}$

Rational

Irrational

$\sqrt{1}$

0.252252225...

$0.\overline{2}$

$\sqrt{100}$

5.5

Identify the numbers as rational or irrational.
If the number is rational, verify it by changing the number into a fraction.

a $0.555555\dots$

.....

b $0.616616661\dots$

.....

c $\sqrt[3]{8}$

.....

d $0.74747474\dots$

.....

Convert decimals to fractions.

Let's convert 0.54 to a fraction.

0	•	$\frac{1}{10}$	$\frac{1}{100}$
0	•	5	4

Use decimal place value

$$0.54 = \frac{\quad}{100} = \frac{\quad}{50}$$

Convert 0.043 to a fraction

$$0.043 = \text{.....}$$

Convert repeating decimals to fractions.

Let's convert $0.\overline{7}$ to a fraction.

Step 1: Set the repeating decimal to

$$x = 0.7777\ldots$$

Step 2: Multiply each side by 10.

$$10x = 7.777\ldots$$

Step 3: Subtract the equations

$$\begin{array}{r} 10x = 7.777\ldots \\ - \quad x = 0.7777\ldots \\ \hline 9 = 7 \end{array}$$

Step 4: Solve for x

$$x = \text{.....}$$

Check your understanding

Identify each number as rational or irrational.

a $\sqrt{22}$

.....

b $9.66666\ldots$

.....

c $8.343343334\ldots$

.....

Why do I need to try this question on my own first?

- To show your tutor what you understand
- To give you more practice
- To show your teacher how you are doing



Rational numbers

Fractions

Terminating decimals

Integers

Whole numbers

Natural
numbers

Repeating decimals

Irrational numbers

Non-repeating, non terminating
decimals

Pi

Square roots of non-
perfect square numbers

Do you have a group of students who need a boost in math?

Each student could receive personalized lessons every week from our specialist one-on-one math tutors.




- ✓ Differentiated instruction for each student
- ✓ Aligned to your state's standards
- ✓ Scaffolded learning to close gaps

“We just had our first session and it went great! The kids really liked it and felt like they were learning! One even said he finally felt like math was making sense.”



Michelle Craig, Instructional Coach,
Sherwood Forest Elementary, Washington

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