



**THIRD SPACE
LEARNING**

Math Intervention Pack

Using the associative property
to multiply

Grade 3

How To Use This Resource

1. Title Slide

Use this slide to activate prior knowledge needed for lesson. Students should be encouraged to initially attempt the question presented independently.

2. Let's Learn

Use this slide to introduce the concept. Tutors should work with the student to explore the concept together, usually using diagrams to support understanding.

3. Follow Me + Your Turn

The tutor should work through the follow me slide, modeling the process and explaining their thinking out loud.

Students should use the your turn slide as an opportunity to work through a question similar to the follow me questions. They should apply the method modeled by the tutor in the follow me slide. Students should be encouraged to explain their thinking out loud.

4. You Do

Students should work through a range of questions that build in complexity.

Tutors can offer support but students should initially be encouraged to attempt these questions independently.

5. Go Further

Use this slide to allow students to apply their understanding to a more challenging question in an unfamiliar context.

6. Support for Slides

The support slide is used to support students during the lesson. In the tutor notes, there will be guidance as to when to use the support slide.

7. Check Your Understanding

Tutors should use this slide to assess the student's knowledge and whether or not they have mastered the concept within the lesson.

Standard

3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication). $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $10 \times 3 = 30$. (Associative property of multiplication). Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property).

Key Mathematical Ideas

1. To develop strategies to solve multiplication facts that are not yet memorized
2. To understand properties of mathematics as proven rules that we follow, that can become tools in solving unknown problems

Overview

Terminology

- **Associative Property of Multiplication:** An extension of the commutative property; to change the order and group two factors to find the convenient products (such as 10) in order to make the multiplication easier. Students may begin to use parentheses at this level. Example: $7 \times 8 \times 5 = 7 \times (8 \times 5) = 7 \times 40 = 280$
- **Commutative Property of Multiplication:** Reversing the order of the factors does not change the product. Example. $8 \times 5 = 40$ and $5 \times 8 = 40$, therefore the product of $8 \times 5 = 5 \times 8$
- **Distributive Property of Multiplication:** Multiplying a sum by a given number is the same as multiplying each addend by the number and then adding the products
- **Product:** The result when two numbers are multiplied
- **Factor:** One of the numbers multiplied to find a product
- **Multiple:** The result of multiplying a whole number by other whole numbers

Overview

Common Misconceptions

Common Misconceptions	Tutoring Strategies	Checks for Understanding
Students going through the motions and not realizing when an order of solving these expressions is easier than others.	The point of the associative property of multiplication is not necessarily to learn that numbers can be multiplied in any order, but to use this as a tool to find the most efficient and simplest ways to multiply numbers—a skill that becomes very important as students move through grade levels. Really bring students attention to this as they go through this lesson so that they are not going through it just to get the answers to the problems, but that they actually look for and try to find the simplest ways of solving the problems. The Let's learn slide really breaks this down for students so spend a good bit of time discussing with them which method they think is the simplest. Make a point to tell them that they can choose to use the method from the start to make it easier for them later.	<ul style="list-style-type: none">• What two factors would be the easiest to start with and why?• Why do we like working with 10 and/or multiples of 10?• Is there a simpler way to solve this problem?• Can you figure out the simplest way to solve this before you even start?

Common Misconceptions	Tutoring Strategies	Checks for Understanding
Students always getting stuck after choosing the first two factors because they end up with a product that they don't know how to multiply the last factor to.	You would most likely see this on the You do slide. Each of the problems on this slide has one way to start that will result in the student getting stuck. The hope is that they won't even choose that route, but if they do, encourage them to go back to the beginning and start differently once they realize they are stuck. Encourage them to think this through the next time before starting at all so that they can avoid getting stuck the next time. Remind them that we don't have to choose the order randomly, we can be intentional about how we start.	<ul style="list-style-type: none">• What two factors would be the easiest to start with and why?• Why do we like working with 10 and/or multiples of 10?• Is there a simpler way to solve this problem?• Can you figure out the simplest way to solve this before you even start?

Title Slide

Cover slide

If students...

- get both sections correct:
 - start at You do
- miss the learning goal section only:
 - start at Let's learn
- miss the prior learning section:
 - start at Prior learning

Prior Learning

If stuck

- For parts b-d, remind students they can build off of facts that they know well to answer the fact they are being asked to solve
- For part e, students can list out 3's facts to help figure out which ones are multiples of 3

If stuck

- Spend some time going over the visuals on this slide to help students see how there are the same number of blocks in each model but they are grouped differently in each example
- If students don't know how to solve 15×2 , remind them that this is the same as 15 twice, or $15 + 15$. Use the blocks to check this
- If students struggled with the first problem, that is actually a good thing. The point of the associative property is to choose the order that is easiest for you—so use the other two examples as options that could have been a better choice at solving the expression than the first method on this slide
- Hopefully students choose the second or third methods on this slide as the simplest method

Questions

- What is 15, twice? (30)
- What do you notice about the final product in each of these examples? (They are all the same)
- Which one do you think is the simplest? (Answers will vary; the second method works with facts students will probably know the best)

Watch out for

- Students not knowing simpler multiplication facts

Follow Me

Modeling prompts

- Let's try multiplying 7×4 first. We know that this equals 28
- The product (the answer to a multiplication problem) in the last problem was 28, so we multiply that by the remaining factor 2. If we don't know what this is by memory, we can double 28 to find the answer. $28 + 28 = 56$ (tutor note: students at this grade level will most likely have no other method to solve 28×2 besides using addition)
- We can check our answer by choosing a different order. Because that was a more difficult order and we got stuck at 28×2 , let's see if we can find a simpler order
- $4 \times 2 = 8$ and $8 \times 7 = 56$
- We got the same answer for both orders, so we know that we did it correctly, and the second method was much easier because we stuck with facts we knew from memory

Your Turn

If stuck

- Let students pick whichever order they want to use for the first part, and challenge them to find an easier order in the second part
- Students should be able to solve all the possible combinations using mental math. If they choose to start with 6×5 and get to 30×2 , they should be able to solve this using multiplication as there is a third grade standard focused on multiplying one-digit numbers by multiples of ten. Even so, they can most likely solve $30 + 30$ using mental math
- Use the Support Slide to help students see the different ways to group the factors in this expression

Questions

- Now that you've tried that order, can you think of an even simpler way to solve this expression? (Answers will vary; if they used the order in the example to the left, an easier order might be $5 \times 2 = 10$ and then $10 \times 6 = 60$. Students may also comment that $6 \times 2 = 12$ and $12 \times 5 = 60$ would be the more difficult choice)

Watch out for

- Students going through the motions and not realizing when an order of solving these expressions is easier than others

Your Turn

If stuck

- Any order that students try is acceptable as long as they arrive at these answers. Encourage them to look for the simplest strategies (they can even solve them mentally if they are able to as long as they say their steps out loud)
- For part a: If students start with 2×8 , they will most likely get stuck on 16×5 . They do not have a strategy to solve this yet in third grade so do not take the time to teach a strategy to them. Instead, encourage them to go back and choose another two factors to start with. You can list them out even and ask them which would be the easiest to start with— 8×5 or 5×2 ? (5×2 would give you 10 and 10 is much easier to multiply by)
- For part b: Same as part a, they will most likely get stuck if they start with 7×2 . Encourage them to find a simpler factor set to start with
- For part c: Same as part a, they will most likely get stuck if they start with 8×2 . Encourage them to find a simpler factor set to start with
- For part d: Same as part a, they will most likely get stuck if they start with 6×4 . Encourage them to find a simpler factor set to start with

Questions

- What two factors do you think would be the easiest to start with and why? (Answers will vary; For example, part a: 2×5 ; part b: 3×2 ; part c: 3×2 ; part d: 5×4)
- Instead of trying to solve this problem that we don't know how to solve, can we go back and find a simpler way to start? (This is if students choose a multiplication equation that they don't know how to solve, for example, part a: if they solve 8×2 first, then they will have to solve 16×5 which they will not know how to do)

Watch out for

- Students always getting stuck after choosing the first two factors because they end up with a product that they don't know how to multiply the last factor to

Your Turn

If stuck

- Make sure students focus on the fact that it's referring to the answer to expressions, not the expressions themselves
- If students aren't sure, have them solve both and compare the answers—if it has to come to this, make sure to remind them of what the associative property of multiplication states

Questions

- Do you agree or disagree with Julian? Why or why not? (Disagree; The Associative Property of Multiplication states that the answer will be the same no matter what order you multiply)
- If you agree: Is there a way we can prove this? ($5 \times 6 = 30$, $30 \times 7 = 210$; $6 \times 5 = 30$, $7 \times 30 = 210$; the answers are the same so we should actually disagree with Julian)
- If you disagree: Can you name a rule in mathematics that explains why he is wrong? (The Associative Property of Multiplication)

Watch out for

- Students thinking that because the ordering is different that the answers will also be different

Support for Slide(s) Your turn

If stuck

- Use the number shapes to help students see the groupings and how both examples have the same total but are grouped differently
- Note that arguably the simplest order is not on this slide, which would be $5 \times 2 = 10$, followed by $10 \times 6 = 60$. You can use this as another example for students

Questions

- What do you notice about the final product in each of these examples? (They are the same)
- Which order do you think is the simplest? (Answers will vary; students may like method 1 better because 12's facts can be hard to remember)

Assessment Question

Correct answers

Answers may vary; Possible answers include:

$$4 \times 5 = 20, 20 \times 3 = 60$$

$$4 \times 3 = 12, 12 \times 5 = 60$$

Students will most likely get stuck if they choose the following order and should choose a different order if they get stuck:

$$3 \times 5 = 15, 15 \times 4 = 60 \text{ (They will not have strategies to use as third graders to solve } 15 \times 4 \text{)}$$

Classroom Tools

Allow students to use the multiplication chart to check their work, or help remind them of a multiplication fact if they forgot it. It should not be used for the entire lesson, as they should try to do as much from memory as they can, or by using another method to problem solve.



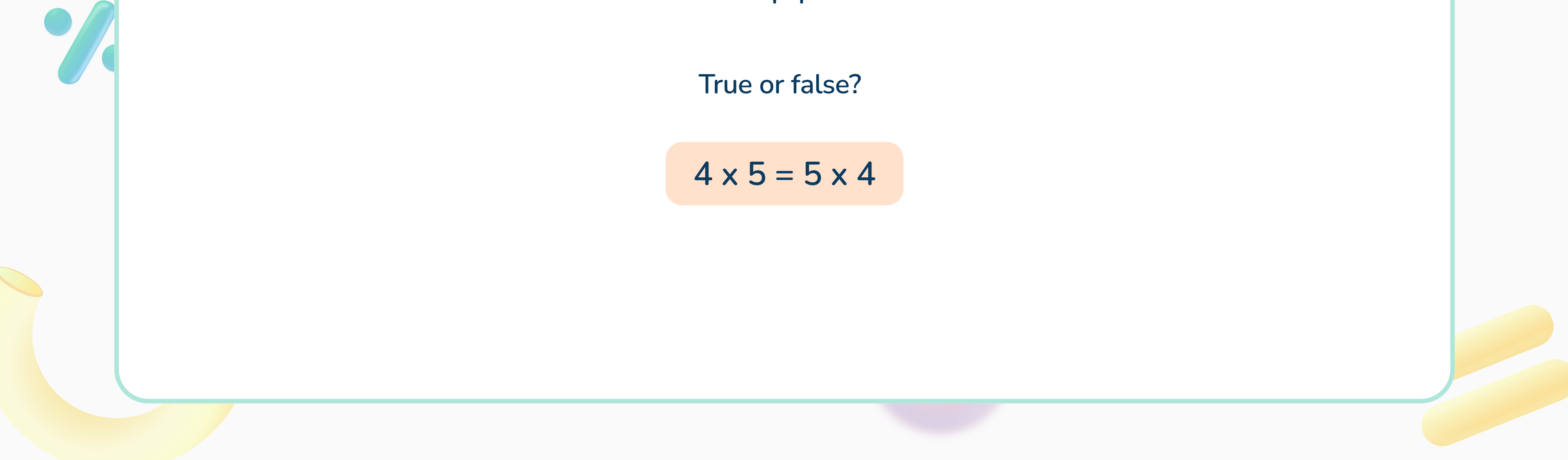
Today you will learn about

Using the associative property to multiply



Warm up question

True or false?

$$4 \times 5 = 5 \times 4$$


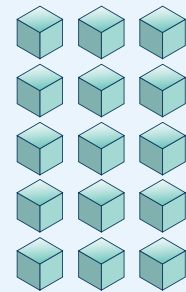
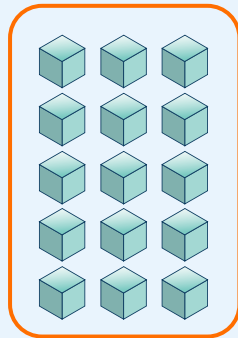
Let's learn

When multiplying three numbers together, it is important to remember that multiplication can be completed in any order.

$$5 \times 3 \times 2$$

Method 1: Multiply in the order given.

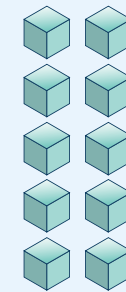
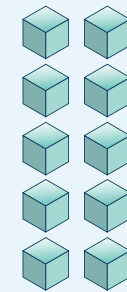
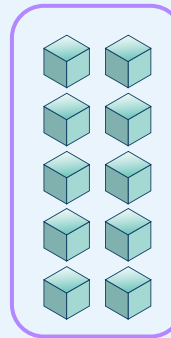
$$5 \times 3 \times 2$$



$$5 \times 3 = \dots \times 2 = \dots$$

Method 2: Swap the order and multiply.

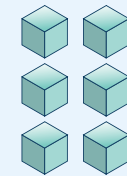
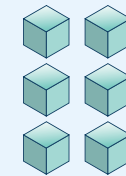
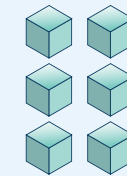
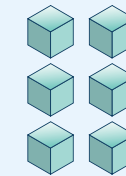
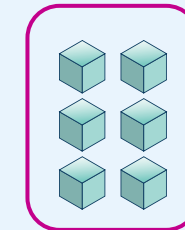
$$5 \times 2 \times 3$$



$$5 \times 2 = \dots \times 3 = \dots$$

Method 3: Choose a different order.

$$3 \times 2 \times 5$$



$$3 \times 2 = \dots \times 5 = \dots$$

Which one do you think is simplest?

The **associative property of multiplication** states that the **product** of three or more numbers remains the same regardless of how the numbers are grouped.

Follow me

- 1 Let's look at this expression.

$$7 \times 4 \times 2$$

- a Decide which part of the expression to complete first.

$$7 \times 4 = \dots\dots\dots$$

- b Then multiply the **product** by the remaining number.

$$\dots\dots\dots \times 2 = \dots\dots\dots$$

The answer can be checked by multiplying in a different order.

c $4 \times 2 = \dots\dots\dots$

d $\dots\dots\dots \times 7 = \dots\dots\dots$

Your turn

- 1 Now try this expression.

$$6 \times 5 \times 2$$

- a Complete one part of the expression.

$$\dots\dots\dots \times \dots\dots\dots = \dots\dots\dots$$

- b Then multiply the **product** by the remaining number.

$$\dots\dots\dots \times \dots\dots\dots = \dots\dots\dots$$

- c Check the answer by multiplying in a different order.

$$\dots\dots\dots \times \dots\dots\dots = \dots\dots\dots$$

$$\dots\dots\dots \times \dots\dots\dots = \dots\dots\dots$$

Complete these problems.

a $2 \times 8 \times 5 = \dots\dots\dots$

b $7 \times 4 \times 2 = \dots\dots\dots$

c $8 \times 3 \times 2 = \dots\dots\dots$

d $5 \times 6 \times 4 = \dots\dots\dots$



Go further



Julian says,
"The answer to $5 \times 6 \times 7$ is different than $6 \times 5 \times 7$."

Do you agree?
Explain your answer.

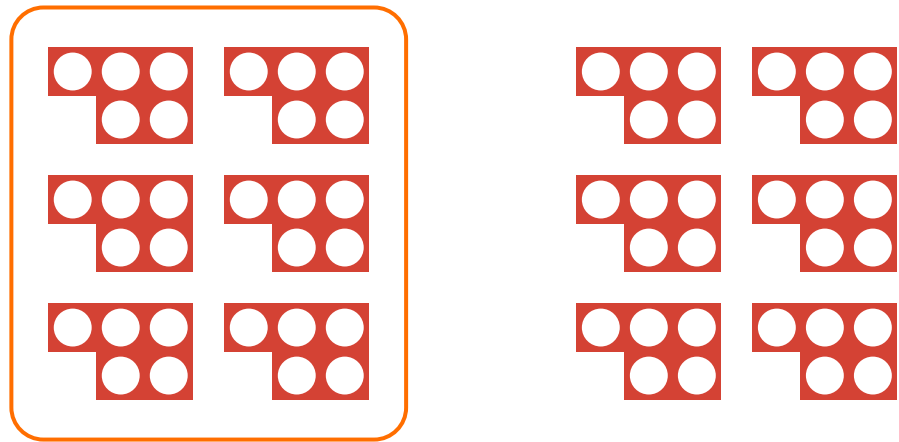
$$6 \times 5 \times 2$$

Let's look at showing this expression using number shapes.

Method 1:

Solve 6×5 (6 groups of 5)

Multiply this by 2.



$$6 \times 5 = \dots\dots\dots$$

$$\dots\dots\dots \times 2 = \dots\dots\dots$$

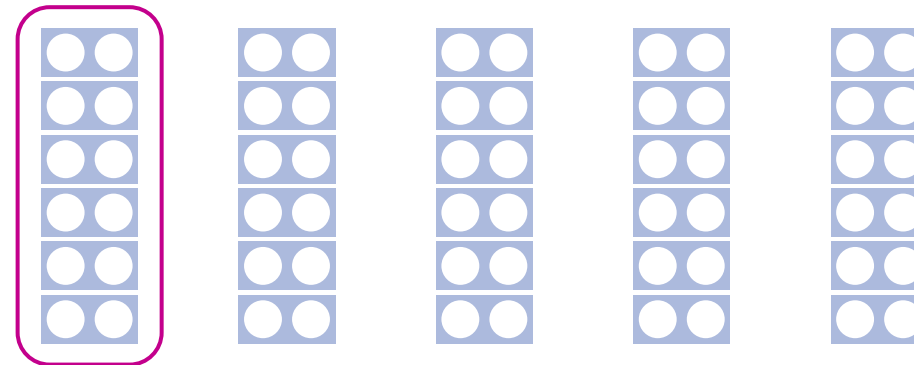
Method 2:

We can change the order we multiply and

solve $6 \times 2 \times 5$

a Find 6 groups of 2.

b Multiply this by 5.



$$6 \times 2 = \dots\dots\dots$$

$$\dots\dots\dots \times 5 = \dots\dots\dots$$

Multiplication is **commutative**, which means it can be completed in any order. The **associative property of multiplication** is an extension of this, stating that we can group factors in any way, and multiply in any order.

Check your understanding

Solve the following expression using the associative property of multiplication:

$$4 \times 3 \times 5$$




Why do I need to try this question on my own first?

- To show your tutor what you understand
- To give you more practice
- To show your teacher how you are doing



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-  Aligned to your state's standards
-  Scaffolded learning to close gaps

“We just had our first session and it went great! The kids really liked it and felt like they were learning! One even said he finally felt like math was making sense.”



Michelle Craig, Instructional Coach,
Sherwood Forest Elementary, Washington

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