Surds

A **surd** is a root that gives an irrational number.

An **irrational number** can't be written as a fraction, and in decimal form is infinitely long with no recurring pattern.

Example $\sqrt{6} \approx 2.4494897$, which is an irrational number.

The square root of 6 is a surd.

THIRD SPACE LEARNING

Multiplying and Dividing Surds

We can multiply and divide surds using the three important **laws of surds**:

$$\sqrt{m} \times \sqrt{n} = \sqrt{mn}$$

$$\sqrt{m} \times \sqrt{m} = m$$

$$\sqrt{m} \div \sqrt{n} = \sqrt{\frac{m}{n}}$$

These are derived from the laws of indices.

Examples

$$\sqrt{5} \times \sqrt{3} = \sqrt{15}$$

If the answer isn't a surd, it should be evaluated and given as an integer.

$$\sqrt{18} \div \sqrt{2} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$3\sqrt{6} \times 2\sqrt{7} = 6\sqrt{42}$$

If there are coefficients in front of the radical (square root) symbol, multiply or divide these separately.

Simplifying Surds

A surd is in its simplest form when the number underneath the square root sign is **as small as possible** - i.e. it has **no square numbers other than 1 as factors**.

The square numbers to 12 are: $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144$

Example Simplify $\sqrt{60}$

1 Find a square number that is a factor of 60

Try to find the **largest square factor** at this stage so that you can simplify the surd fully in one go.

4 is a square number and a factor of 60

$$4 \times 15 = 60$$

2 Rewrite the surd as a product of these two factors: $\sqrt{60} = \sqrt{4} \times \sqrt{15}$

3 Evaluate the root of the square number: $\sqrt{60} = 2 \times \sqrt{15} = 2\sqrt{15}$

This is fully simplified because no square numbers (other than 1) are factors of 15. If this was not the case, you would need to repeat the process.

THIRD SPACE LEARNING

Adding and Subtracting Surds

To add or subtract surds, they must be **like surds** - the numbers underneath the square root signs must be the same.

Example $\sqrt{3}+\sqrt{3}+2\sqrt{3}=4\sqrt{3}$

All of these surds contain $\sqrt{3}$
They are **like surds** so can be added together.

Example $8\sqrt{5}+4\sqrt{3}-2\sqrt{5}=6\sqrt{5}+4\sqrt{3}$

Here we can only combine the surds containing $\sqrt{5}$ because $\sqrt{5}$ and $\sqrt{3}$ are not like surds.

If the surds are not like surds initially, simplifying one or both may enable you to convert them to like surds and then add or subtract.

Example $\sqrt{24}+3\sqrt{6}$

1 Simplify $\sqrt{24}$: $\sqrt{24}=\sqrt{4}\times\sqrt{6}$

$$=2\times\sqrt{6}=2\sqrt{6}$$

2 Combine like surds:

$$2\sqrt{6}+3\sqrt{6}=5\sqrt{6}$$

THIRD SPACE LEARNING

Rationalise the Denominator

To **rationalise the denominator** we convert the denominator of a fraction from a surd to a rational number.

Denominator is a single surd

Example Rationalise the denominator: $\frac{8}{\sqrt{2}}$

1 Multiply the numerator and denominator by the **surd in the denominator**.

$$\frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

We use the law $\sqrt{m} \times \sqrt{m} = m$ to change the denominator to a rational number. $\sqrt{2} \times \sqrt{2} = 2$

2 Simplify the expression fully:

$$\frac{8\sqrt{2}}{2} \begin{matrix} \div 2 \\ \\ \div 2 \end{matrix} = 4\sqrt{2}$$

We **only** cancel factors from coefficients and not from inside square root symbols.

Denominator is a surd expression

Example Rationalise the denominator: $\frac{4}{5 + \sqrt{2}}$

1 Change the sign of the expression in the denominator. $5 - \sqrt{2}$

2 Multiply numerator and denominator by **this expression**.

$$\frac{4 \times (5 - \sqrt{2})}{5 + \sqrt{2} \times (5 - \sqrt{2})}$$

We use the difference of two squares to eliminate all surds from the denominator.

3 Simplify: $\frac{20 - 4\sqrt{2}}{25 + \cancel{5\sqrt{2}} - \cancel{5\sqrt{2}} - 2} = \frac{20 - 4\sqrt{2}}{23}$

Surd expressions in denominator cancel out.

THIRD SPACE LEARNING