

# Factors

A **factor** of an integer divides that integer exactly with **no remainder**.

Whole number

 **Example**

3 is a factor of 18 because  $18 \div 3 = 6$

The result of this division is an integer, so 3 and 6 are factors of 18

4 is **not** a factor of 18 because  $18 \div 4 = 4.5$

The result of this division is **not** an integer, so 4 is **not** a factor of 18

To find all of the factors of an integer, we write out all of the **factor pairs** in order.

 **Example**

Find all of the factors of 24

1 Begin with 1 and the number itself. Every integer will have a pair of factors like this (1 and the number itself).

$$1 \times 24$$

2 Work in order. 2 is a factor because  $24 \div 2 = 12$ . This divides exactly so 2 and 12 are factors of 24

$$2 \times 12$$

3 Keep working in order - we check 3 and 4. They are both factors of 24, so we write down those factor pairs.

$$3 \times 8$$

$$4 \times 6$$

4 5 is not a factor of 24 because  $24 \div 5 = 4.8$  (not an integer)

5 The next number we would check is 6, but we already have 6 in the list, so we have found all of the factors.

The factors of 24 are:

1, 2, 3, 4, 6, 8, 12, 24

# Multiples

A **multiple** of a number is the result of multiplying that number by an integer.

 **Example**

The first five multiples of 7 are: 7, 14, 21, 28 and 35

These are the answers to:  
 $1 \times 7$ ,  $2 \times 7$ ,  $3 \times 7$  ...

There are an **infinite number** of multiples of a number - we cannot list them all. However, we can find a particular multiple of a number quite easily.

 **Example**

Find the 9<sup>th</sup> multiple of 4

$$9 \times 4 = 36$$

# Prime Factors

**Prime factors** are prime numbers that are factors of another number.

A prime number has exactly two factors. These are 1 and the number itself.

Any number that is not prime (or 1) is **composite** - it can be written as a product of two or more **prime factors**.

 **Example**

$$6 = 2 \times 3$$


Composite number

Prime factors

 **Example**

Express the product of prime factors in **index form**

$$\begin{aligned} 60 &= 2 \times 2 \times 3 \times 5 \\ &= 2^2 \times 3 \times 5 \end{aligned}$$

 **Example**

$$24 = 2 \times 12$$

12 is **not** a prime number, so we break it down further.

2 is a prime number.

$$= 2 \times 2 \times 6$$

6 is **not** prime so break it down further.

Prime factors

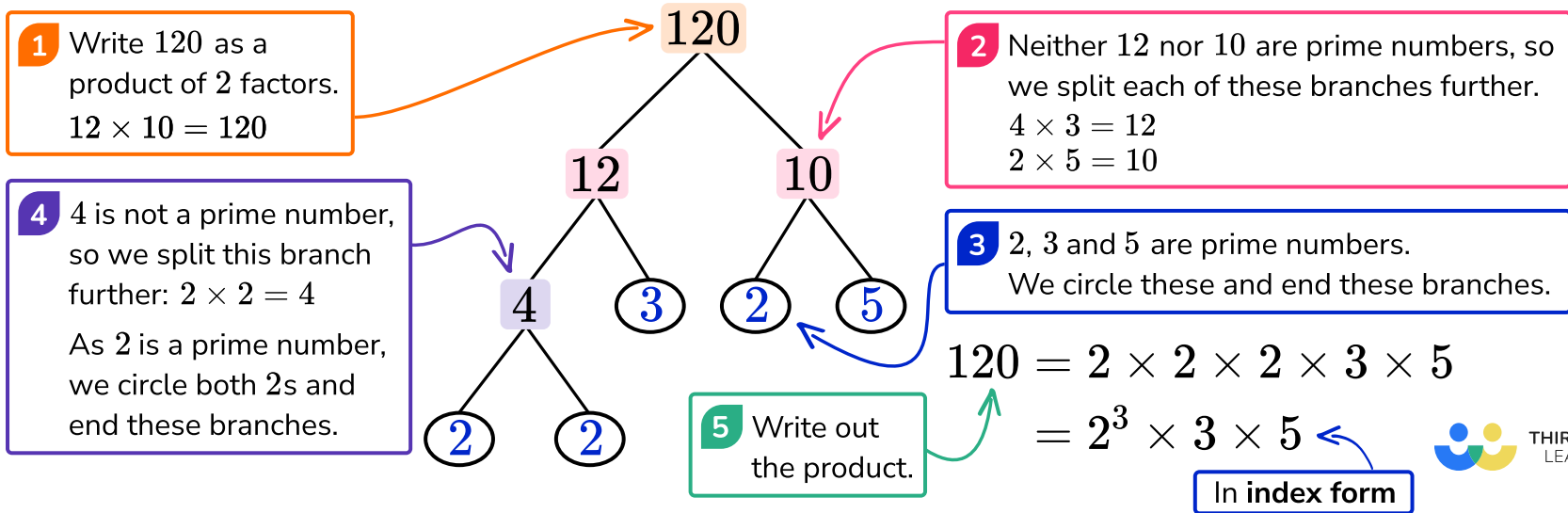
$$= 2 \times 2 \times 2 \times 3$$

All prime factors

# Factor Trees

A factor tree is used to find factors of a number and express that number as a product of primes. Each branch is split into factors and ends on a prime number.

 **Example** Write 120 as a product of prime factors.



# Highest Common Factor

The highest common factor (HCF) or greatest common factor is the **largest integer** (whole number) that two or more numbers can both be divided by.

## By listing factors (small numbers)

 **Example** Find the HCF of 12 and 18

**1** List the factors of each number in order:

Factors of 12: 1, 2, 3, 4, **6**, 12

Factors of 18: 1, 2, 3, **6**, 9, 18

**6** is the **highest** number that is in **both** lists.

**2** Select the **highest** number that is **common** to both lists. HCF of 12 and 18 = 6

## Using product of primes (large numbers)

 **Example** Find the HCF of 60 and 96

**1** Write both numbers as a product of primes:

$$60 = 2 \times 2 \times 3 \times 5$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

You can use a **factor tree** to help you with this step.

**2** Find prime factors that are **common** to both lists.

**3** The HCF is the product of the common factors.

$$2 \times 2 \times 3 = 12$$

# Lowest Common Multiple

The lowest common multiple (LCM) is the **smallest integer** (whole number) that belongs to the multiplication table of two or more numbers.

## By listing multiples (small numbers)

 **Example** Find the LCM of 4 and 6

**1** List the multiples of each number in order:

Multiples of 4: 4, 8, **12**, 16, 20, ...

Multiples of 6: 6, **12**, 18, 24, 30, ...

**12** is the  
first number  
that is in  
both lists.

**2** Select the **lowest** number that is **common** to both lists. LCM of 4 and 6 = 12

## Using product of primes (large numbers)

 **Example** Find the LCM of 24 and 90

**1** Write both numbers as a product of primes:

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$$90 = 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

You can use a  
**factor tree** to  
help you with  
this step.

**2** Select the **largest power** of **every prime factor**.

**3** The LCM is the product of these factors:

$$2^3 \times 3^2 \times 5 = 360$$