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LEARNING

# GCSE Maths Intervention Pack

Using Circle Theorems

Grade 7

## Teacher Notes

### Question Sets

#### Set 1: Angles in segments and at the centre

Calculate missing angles using the two circle theorems:

“Angles in the same segment are equal”

“The angle at the centre is twice the angle at the circumference”

Key words: Centre, circumference, radius, segment

#### Set 2: Angles in semicircles and cyclic quadrilaterals

Calculate missing angles using the two circle theorems:

“Angles in a semicircle are  $90^\circ$ ”

“The sum of opposite angles in a cyclic quadrilateral is  $180^\circ$ ”

Key words: Centre, circumference, cyclic quadrilateral, diameter, semicircle

#### Set 3: Tangents and alternate segments

Calculate missing angles using the three circle theorems:

“The angle between the tangent and the radius is  $90^\circ$ ”

“Two intersecting tangents are the same length”

“The Alternate Segment Theorem”

Key words: Centre, circumference, perpendicular, radius, segment, tangent



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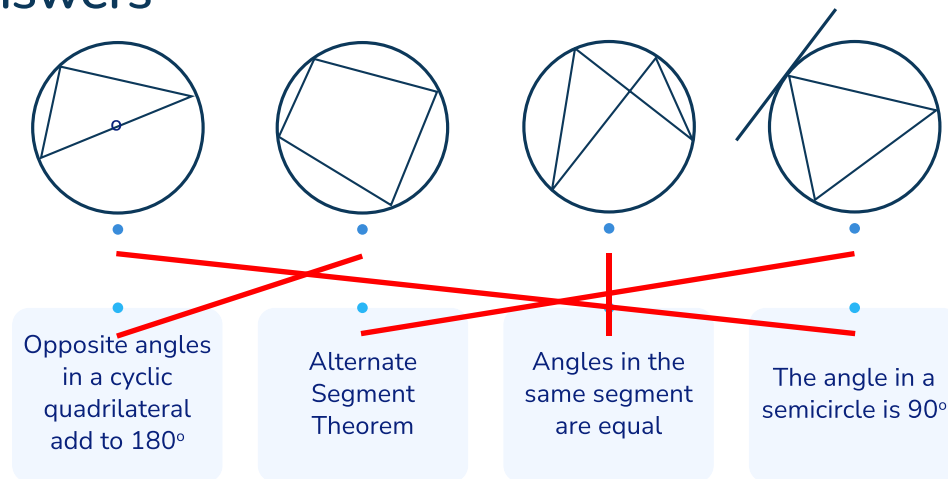
**Gabriel Ogbeifun,**  
Head of Mathematics, Regent High School

## Slide 1: Cover Slide

### Teaching Prompts

- Can you match any keywords with a diagram?

### Answers



### Teacher Reference Only

Training video

### Common Misconceptions

- (Angle at the centre is twice the angle at the circumference) Halving and doubling incorrectly.
- (Angles in a semicircle are  $90^\circ$ ) Incorrectly identifying parts of a circle, specifically a diameter or a chord.
- (Types of triangle, and the length of tangents) Incorrect assumption of isosceles triangles (usually only from two radii or two tangents meeting at a point)
- (Alternate segment theorem) The incorrect angle is stated.
- (Angles in the same segment are equal) Students assume lines for being parallel and therefore apply “alternate angles” to the problem.
- Mixing up the circle theorems.

### Terminology

- Segment: The area between a chord and the connecting arc of a circle.
- Cyclic quadrilateral: A quadrilateral with all four vertices on the circumference of a circle.
- Tangent: A line that touches a curve at a point.

## Slide 2: Try this exam-style question

### Set 1: Angles in segments and at the centre.

#### Teaching Prompts

- Can you try this question by yourself?
- 

#### If Stuck

- Move on to the next slide.
- 

#### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle ABD as  $47^\circ$ .
- a) (1 mark) for indicating angle ACD as  $47^\circ$ .
- a) (1 mark) for reason "angles in the same segment are equal"

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle AOC as  $96^\circ$ .
  - b) (1 mark) for indicating angle OAC as  $42^\circ$ .
  - b) (1 mark) for reason "angle at the centre is twice the angle at the circumference"
  - b) (1 mark) for reason "base angles of an isosceles triangle are equal"
- 

#### Watch out for

- (Angle at the centre is twice the angle at the circumference) Halving and doubling incorrectly.
- (Types of triangle) Incorrect assumption of isosceles triangles (usually only from two radii)
- Mixing up the circle theorems.

## Slides 3 and 4: Let's go through it together...

Set 1: Angles in segments and at the centre.

### Teaching Prompts

- Can you match the diagrams with the circle theorems?
  - Can you find any angles in the diagram using rules about triangles or opposite angles?
- 

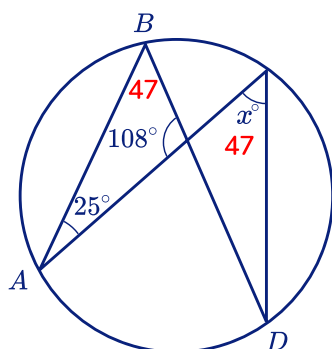
### Answers

1)

Part (a) involves angles in the same segment.

Part (b) involves the angle at the centre theorem.

2)



$$\angle ABD = 47^\circ$$

Reason: Angle in a triangle add to  $180^\circ$

---

3)

$$x = 47^\circ$$

Reason: Angles in the same segment are equal

---

### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle ABC as  $47^\circ$ .
- a) (1 mark) for indicating angle ACD as  $47^\circ$ .
- a) (1 mark) for reason "angles in the same segment are equal"

## Slide 5: Let's go through it together...

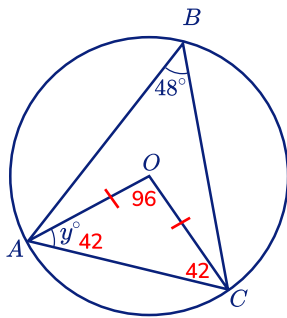
Set 1: Angles in segments and at the centre.

### Teaching Prompts

- Can you see an isosceles triangle? (triangle AOC)
- What circle theorem will we need for this question? (The angle at the centre theorem)

### Answers

4)



$$\angle AOC = 96^\circ$$

Reason: The angle at the centre is twice the angle at the circumference

5)

$$y = 42$$

Reason: Base angles of an isosceles triangle are equal

### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle AOC as  $96^\circ$ .
- b) (1 mark) for indicating angle OAC as  $42^\circ$ .
- b) (1 mark) for reason "angle at the centre is twice the angle at the circumference"
- b) (1 mark) for reason "base angles of an isosceles triangle are equal"

## Slide 6: Your turn...

Set 1: Angles in segments and at the centre.

### Teaching Prompts

- Can you see what circle theorems apply to this diagram?
- 

### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle AOC as  $96^\circ$ .
- a) (1 mark) for indicating angle ABD as  $48^\circ$ .
- a) (1 mark) for reason "angle at the centre is twice the angle at the circumference"
- a) (1 mark) for reason "base angles of an isosceles triangle are equal"

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle ACD as  $48^\circ$ .
- b) (1 mark) for reason "angles in the same segment are equal"

## Slide 7: Try this exam-style question...

### Set 2: Angles in semicircles and cyclic quadrilaterals.

#### Teaching Prompts

- Can you try this question by yourself?
- 

#### If Stuck

- Move on to the next slide.
- 

#### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle ABC as  $90^\circ$ .
- a) (1 mark) for indicating angle BAC as  $32^\circ$ .
- a) (1 mark) for reason "angle in a semicircle is  $90^\circ$ "

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle ABC as  $99^\circ$ .
  - b) (1 mark) for reason "opposite angles in a cyclic quadrilateral add to  $180^\circ$ "
- 

#### Watch out for

- (Angles in a semicircle are  $90^\circ$ ) Incorrectly identifying parts of a circle, specifically a diameter or a chord.
- Mixing up the circle theorems.

## Slides 8 and 9: Let's go through it together...

Set 2: Angles in semicircles and cyclic quadrilaterals.

### Teaching Prompts

- Can you match the diagrams to the circle theorems?
- a) Can you label a right angle? (angle ABC)
- b) What is the circle theorem involving cyclic quadrilaterals? (opposite angles add to 180 degrees)

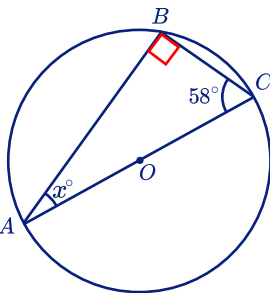
### Answers

1)

Part (a) involves the angle in a semicircle.

Part (b) involves a cyclic quadrilateral.

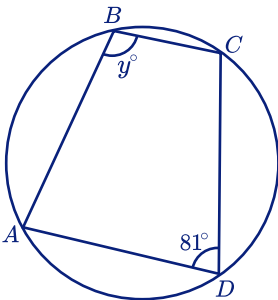
2)



$$\angle ABC = 90^\circ$$

Reason: The angle in a semicircle is  $90^\circ$

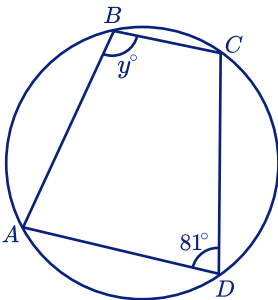
3)



$$x = 32^\circ$$

Reason: Angles in a triangle added to  $180^\circ$

4)



$$y = 99^\circ$$

Reason: opposite angles in a cyclic quadrilateral add to  $180^\circ$

### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle ABC as  $90^\circ$ .
- a) (1 mark) for indicating angle BAC as  $32^\circ$ .
- a) (1 mark) for reason "angle in a semicircle is  $90^\circ$ "

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle ABC as  $99^\circ$ .
- b) (1 mark) for reason "opposite angles in a cyclic quadrilateral add to  $180^\circ$ "

## Slide 10: Your turn...

Set 2: Angles in semicircles and cyclic quadrilaterals.

### Teaching Prompts

- Can you see a right angle?
  - Can you see a cyclic quadrilateral?
- 

### Mark Scheme

- a) (1 mark) for indicating angle ACD as  $90^\circ$ .
- a) (1 mark) for indicating angle CAD as  $36^\circ$ .
- a) (1 mark) for reason "angle in a semicircle is  $90^\circ$ "
- b) (1 mark) for indicating angle ABC as  $126^\circ$ .
- b) (1 mark) for reason "opposite angles in a cyclic quadrilateral add to  $180^\circ$ "

## Slide 11: Try this exam-style question...

### Set 3: Tangents and Alternate Segments.

#### Teaching Prompts

- Can you try this question by yourself?
- 

#### If Stuck

- Move on to the next slide.
- 

#### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle OAC or OBC as  $90^\circ$ .
- a) (1 mark) for indicating angle AOC as  $62^\circ$ .
- a) (1 mark) for reason "The angle between a tangent and radius is  $90^\circ$ "

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle DAB as  $65^\circ$ .
  - b) (1 mark) for reason "alternate segment theorem"
- 

#### Watch out for

- (Angles in a semicircle are  $90^\circ$ ) Incorrectly identifying parts of a circle, specifically a diameter or a chord.
- (Types of triangle, and the length of tangents) Incorrect assumption of isosceles triangles (usually only from two radii or two tangents meeting at a point)
- (Alternate segment theorem) The incorrect angle is stated.
- Mixing up the circle theorems.

## Slides 12 and 13: Let's go through it together...

Set 3: Tangents and Alternate Segments.

### Teaching Prompts

- Can you match the diagrams to the circle theorems?
- 

### Answers

- 1)  
Part (a) involves the angle between a tangent and radius is  $90^\circ$  and tangents which meet at the same point are equal in length.  
Part (b) involves The Alternate Segment Theorem.

2)  $\angle OAC = 90^\circ$

Reason: The angle between a tangent and radius is  $90^\circ$

---

3)  $\angle OCA$

4)  $x = 62^\circ$

Reason: Angles in a triangle add to  $180^\circ$

---

5)  $y = 65^\circ$

Reason: Alternate segment theorem

---

### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle OAC or OBC as  $90^\circ$ .
- a) (1 mark) for indicating angle AOC as  $62^\circ$ .
- a) (1 mark) for reason "The angle between a tangent and radius is  $90^\circ$ "

Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle DAB as  $65^\circ$ .
- b) (1 mark) for reason "alternate segment theorem"

## Slide 14: Your turn...

Set 3: Tangents and Alternate Segments.

### Teaching Prompts

- Can you label any right angles?
  - Is there an isosceles triangle?
  - Can you use the alternate segment theorem?
- 

### Mark Scheme

Allow full marks for any other correct method that leads to final answer

- a) (1 mark) for indicating angle OAD as  $90^\circ$ .
- a) (1 mark) for indicating angle CAD as  $58^\circ$ .
- a) (1 mark) for reason "The angle between a tangent and radius is  $90^\circ$ "
- a) (1 mark) for indicating angle CAO as  $32^\circ$ .

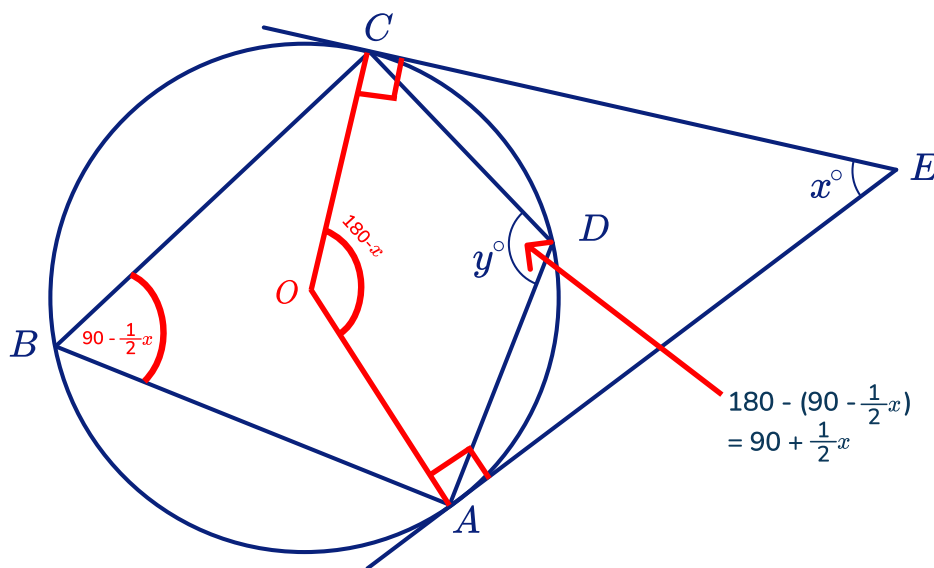
Allow full marks for any other correct method that leads to final answer

- b) (1 mark) for indicating angle ABC as  $58^\circ$ .
- b) (1 mark) for reason "alternate segment theorem"

## Slide 15: Ready for a Challenge?

### Teaching Prompts

- Start by adding the centre point  $O$  to the circle and radii  $OC$  and  $OA$ .
- Can you label any right angles?
- Can you find the angle  $AOC$ ?
- Can you use a circle theorem to find angle  $ABC$  in terms of  $x$ ?



### Mark Scheme

Allow full marks for any other correct method that leads to final answer

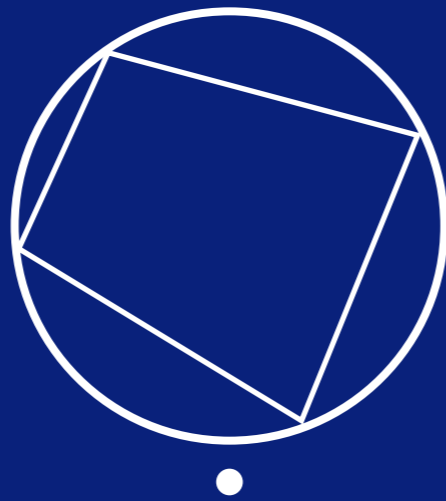
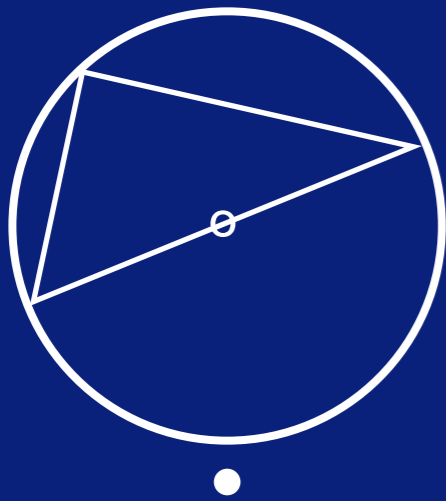
- (1 mark) for adding radii to the diagram at  $C$  and  $A$  and labelling right angles
- (1 mark) for finding angle  $AOC$  as  $180 - x$
- (1 mark) for finding angle  $ABC$  as  $\frac{180-x}{2} = 90 - \frac{1}{2}x$
- (1 mark) for angle  $ADC$  as  $180 - (90 - \frac{1}{2}x)$
- (1 mark) for finding angle  $ADC$  as  $90 + \frac{1}{2}x$

## Slide 16: What have we learnt?

- Can you see where the student has gone wrong? (they have assumed  $x$  is the same as angle OAC )
- What should they have done instead? (used the isosceles triangle OAC to find angle AOC as  $88^\circ$ , then halved that to give angle ABC as  $44^\circ$ )
- Can you see where the student has gone wrong? (they have confused the alternate segment theorem with alternate angles in parallel lines)
- What should they have done instead? ( $y = 55^\circ$ )

# Using Circle Theorems

Match the diagram to the circle theorem



Opposite angles  
in a cyclic  
quadrilateral  
add to  $180^\circ$

Alternate  
Segment  
Theorem

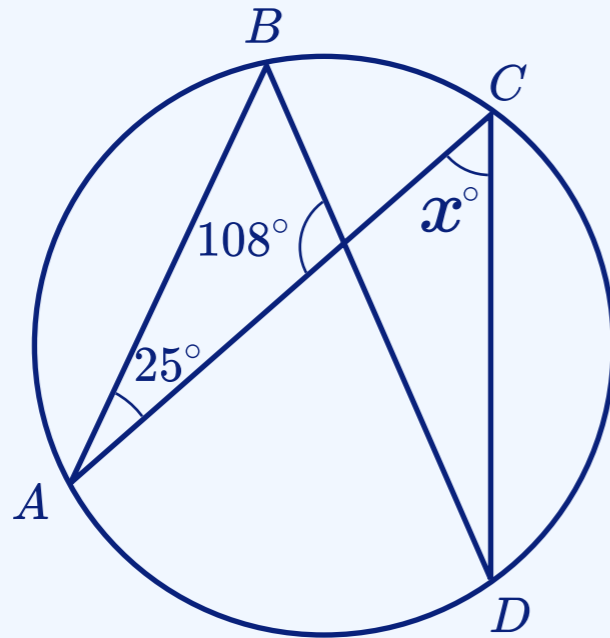
Angles in the  
same segment  
are equal

The angle in a  
semicircle is  $90^\circ$

Try this exam-style question...

a) Find the value of  $x$ .

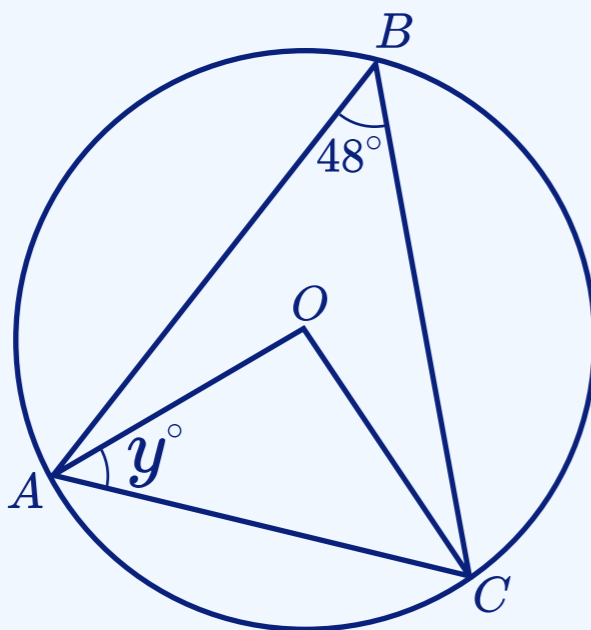
Give reasons for your answer.



(3)

b) Find the value of  $y$ .

Give reasons for your answer.

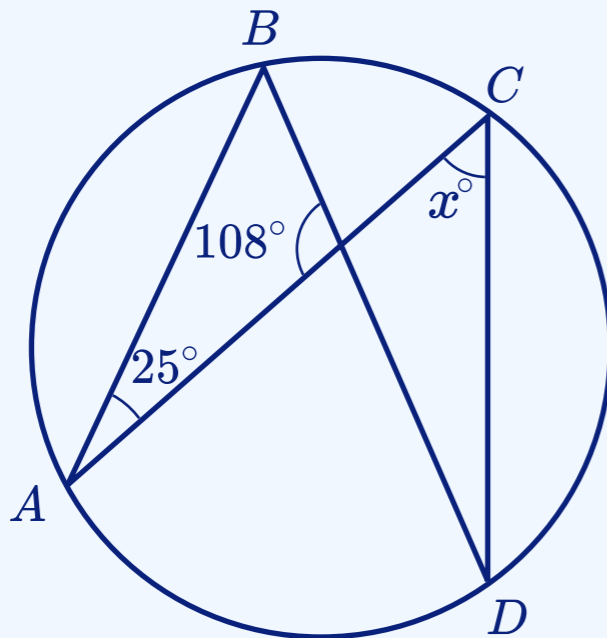


(4)

## Let's go through it together...

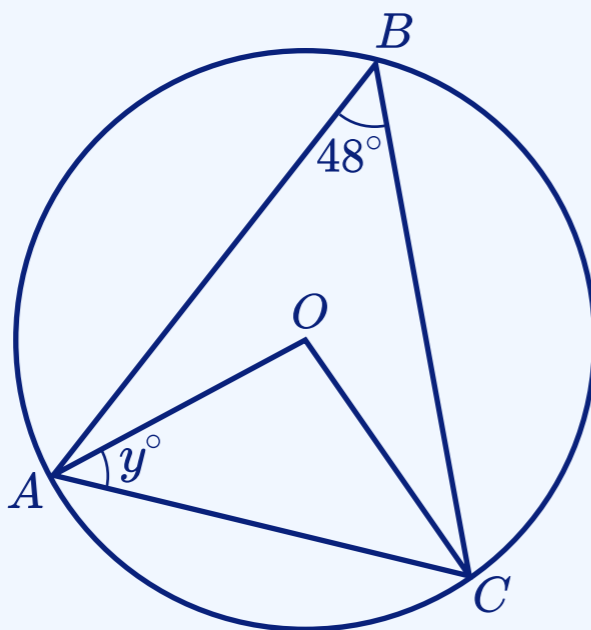
a) Find the value of  $x$ .

Give reasons for your answer.



b) Find the value of  $y$ .

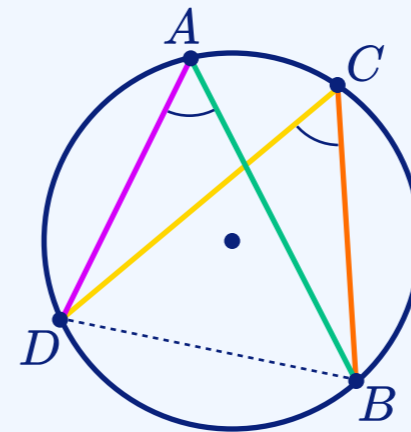
Give reasons for your answer.



We can calculate missing angles in circles using **circle theorems**.

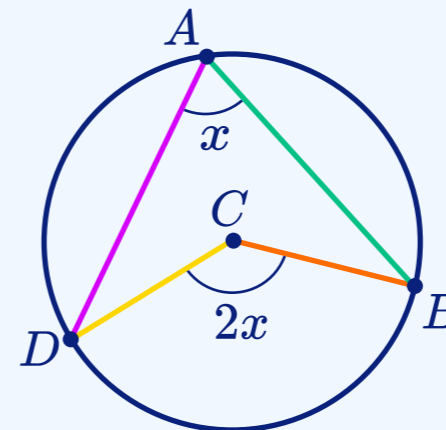
Here are two circle theorems.

### Angles In The Same Segment Theorem



Angles in the same segment are equal.

### Angle At The Centre Theorem



The angle at the centre is twice the angle at the circumference.

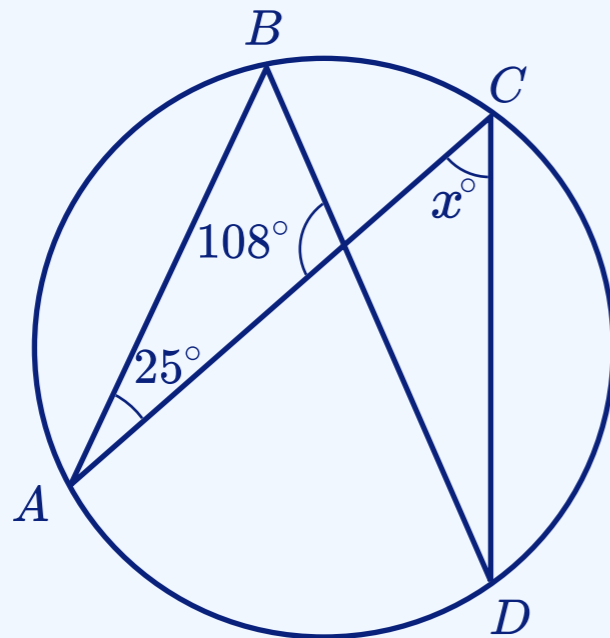
We also need to look for **isosceles triangles** formed from two radii of the circle.

1 Can you match the diagrams in the questions to the circle theorem?

## Let's go through it together...

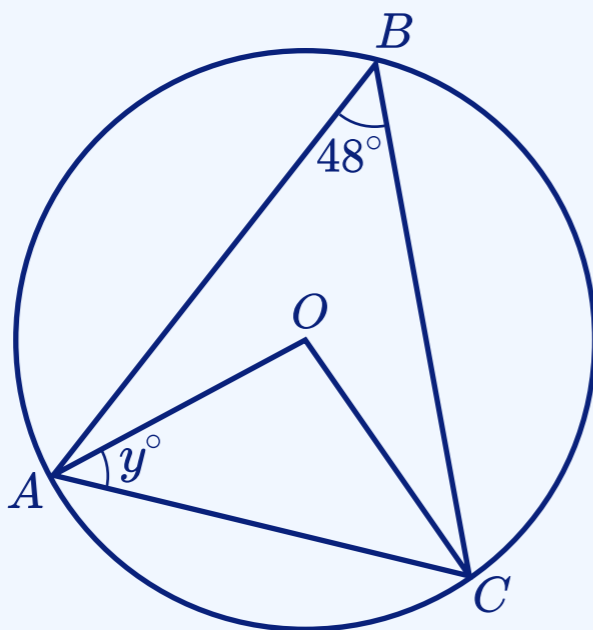
a) Find the value of  $x$ .

Give reasons for your answer.



b) Find the value of  $y$ .

Give reasons for your answer.



As well as calculating missing angles, we need to **give reasons** for the answer.

We need to use phrases like:

“Angles in the same segment are equal.”

“The angle at the centre is twice the angle at the circumference.”

We may also need to use other angle facts and state the reasons.

2 Find the size of  $\angle ABD$  and state the reason.

$$\angle ABD = \dots\dots\dots$$

Reason:

.....  
 .....

3 Find the size of  $x$  and state the reason.

$$x = \dots\dots\dots$$

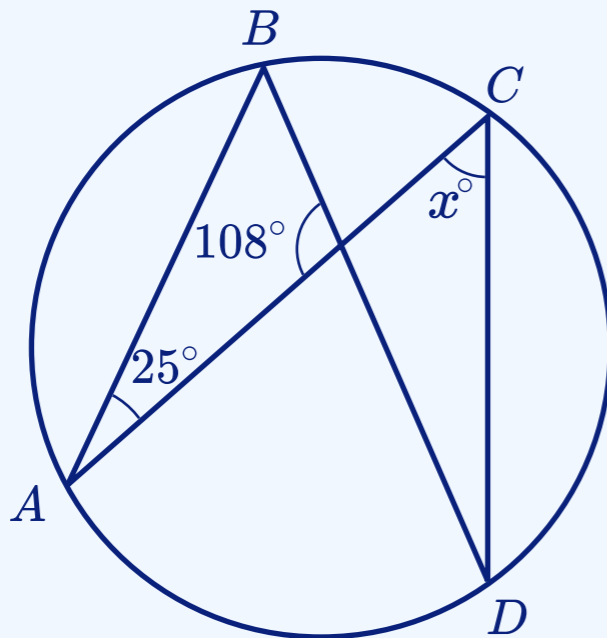
Reason:

.....  
 .....

## Let's go through it together...

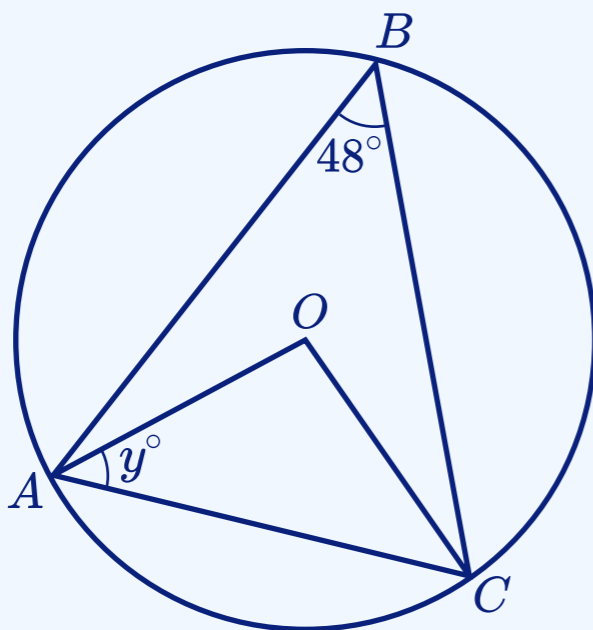
a) Find the value of  $x$ .

Give reasons for your answer.



b) Find the value of  $y$ .

Give reasons for your answer.



As well as calculating missing angles, we need to **give reasons** for the answer.

We need to use phrases like:

“Angles in the same segment are equal.”

“The angle at the centre is twice the angle at the circumference.”

We may also need to use other angle facts and state the reasons.

4 Find the size of  $\angle AOC$  and state the reason.

$$\angle AOC = \dots\dots\dots$$

Reason:

.....  
.....

5 Find the size of  $y$  and state the reason.

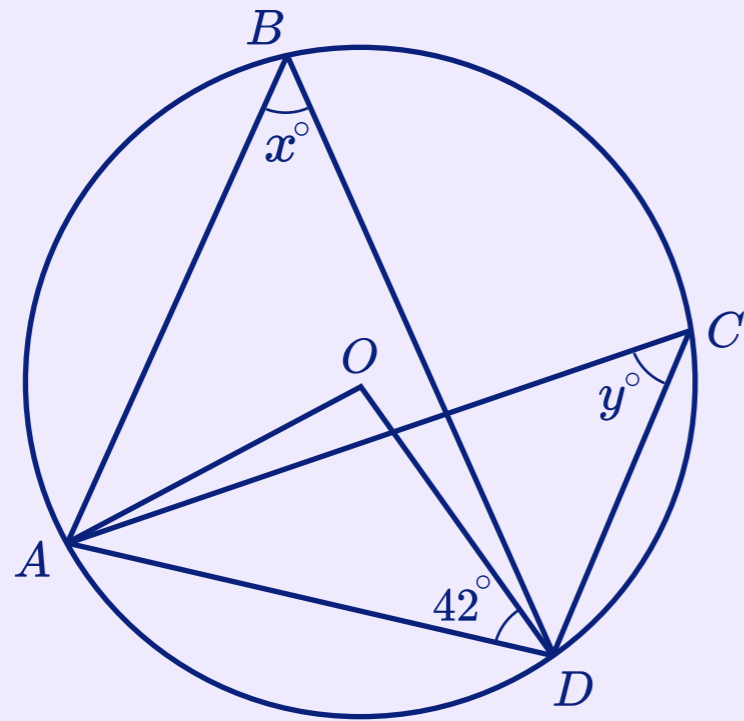
$$y = \dots\dots\dots$$

Reason:

.....  
.....

## Your turn...

The points  $A$ ,  $B$ ,  $C$  and  $D$  are on the circumference of a circle centre  $O$ .



a) Find the value of  $x$ .

Give reasons for your answer.

(4)

b) Find the value of  $y$ .

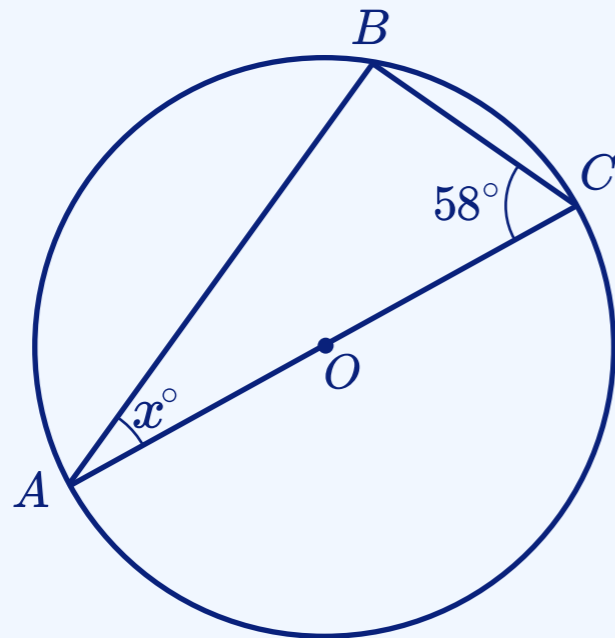
Give reasons for your answer.

(2)

Try this exam style question...

a) Find the value of  $x$ .

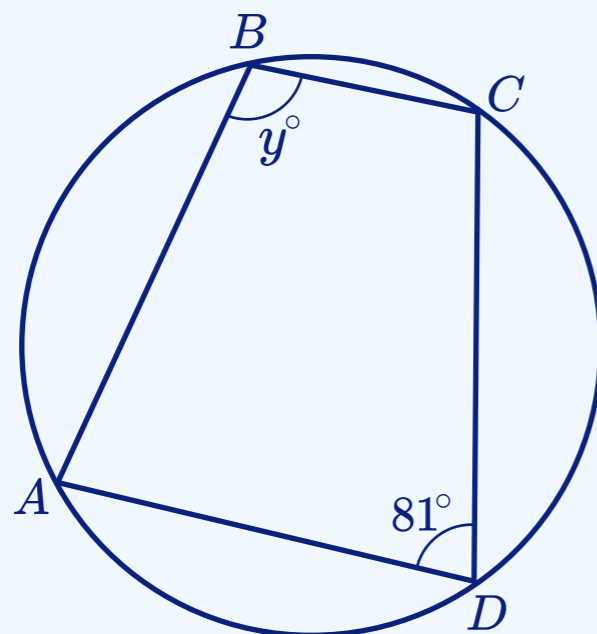
Give reasons for your answer.



(3)

b) Find the value of  $y$ .

Give reasons for your answer.

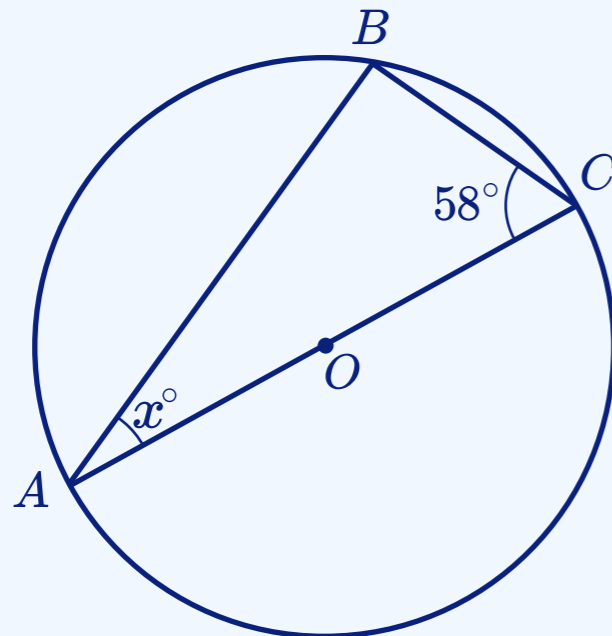


(2)

## Let's go through it together...

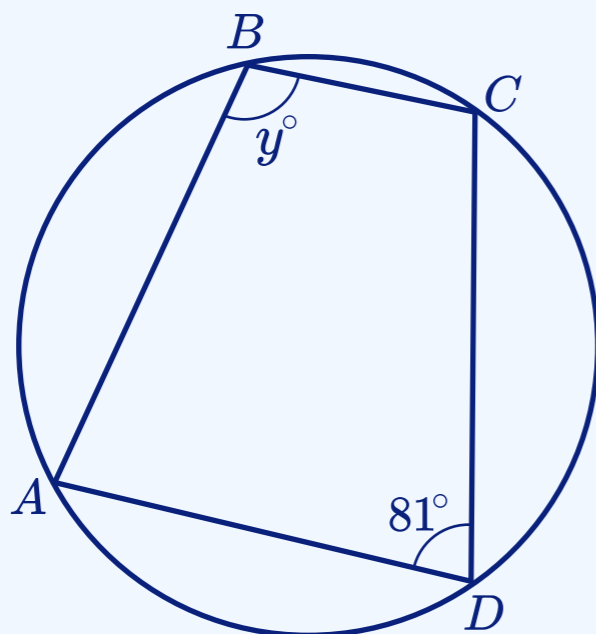
a) Find the value of  $x$ .

Give reasons for your answer.



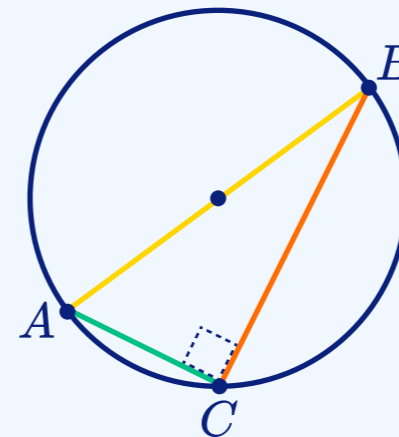
b) Find the value of  $y$ .

Give reasons for your answer.



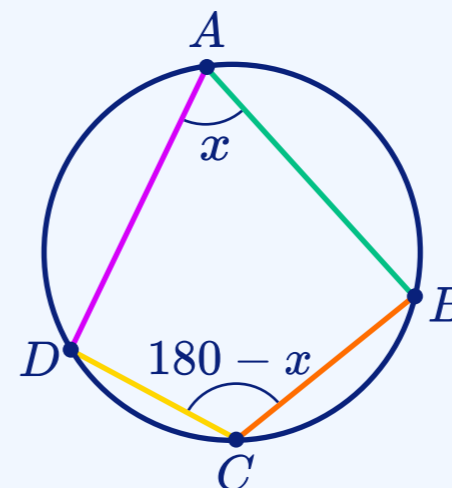
Here are two more circle theorems.

### Angles In A Semicircle



The angle in a semicircle is 90 degrees.

### Cyclic Quadrilateral



The opposite angles in a cyclic quadrilateral total 180 degrees.

(3)

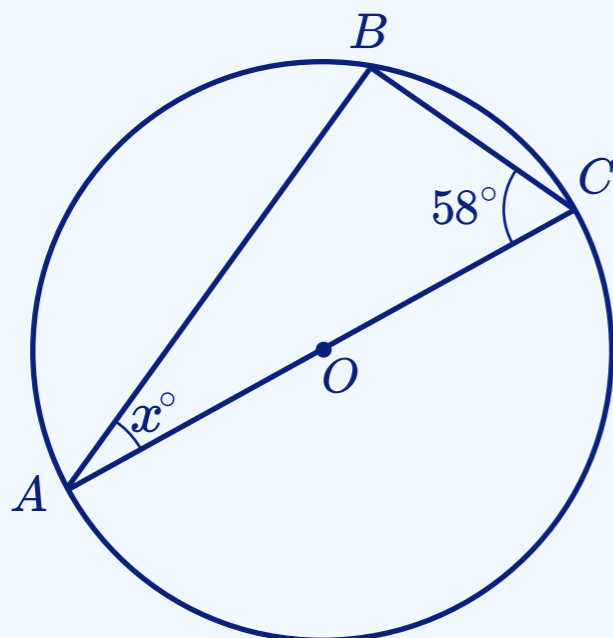
- 1 Can you match the diagrams in the questions to the circle theorem?

(2)

## Let's go through it together...

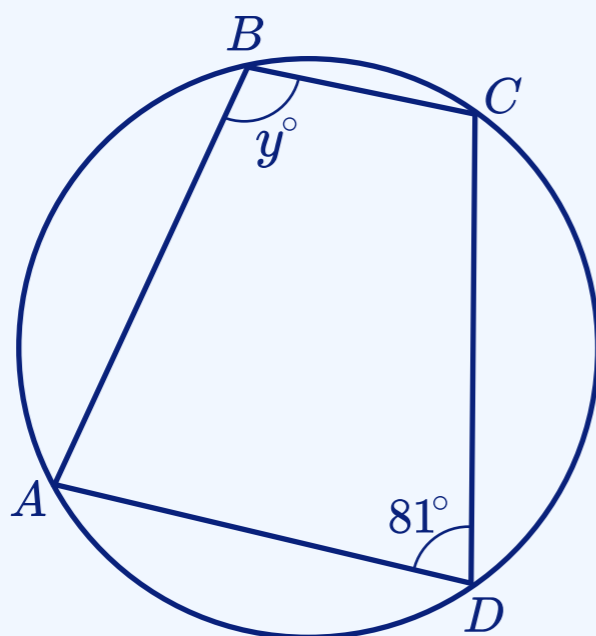
a) Find the value of  $x$ .

Give reasons for your answer.



b) Find the value of  $y$ .

Give reasons for your answer.



2 Find the size of  $\angle ABC$  and state the reason.

$$\angle ABC = \dots\dots\dots$$

Reason:

.....  
 .....

3 Find the size of  $x$  and state the reason.

$$x = \dots\dots\dots$$

Reason:

.....  
 .....

4 Find the size of  $y$  and state the reason.

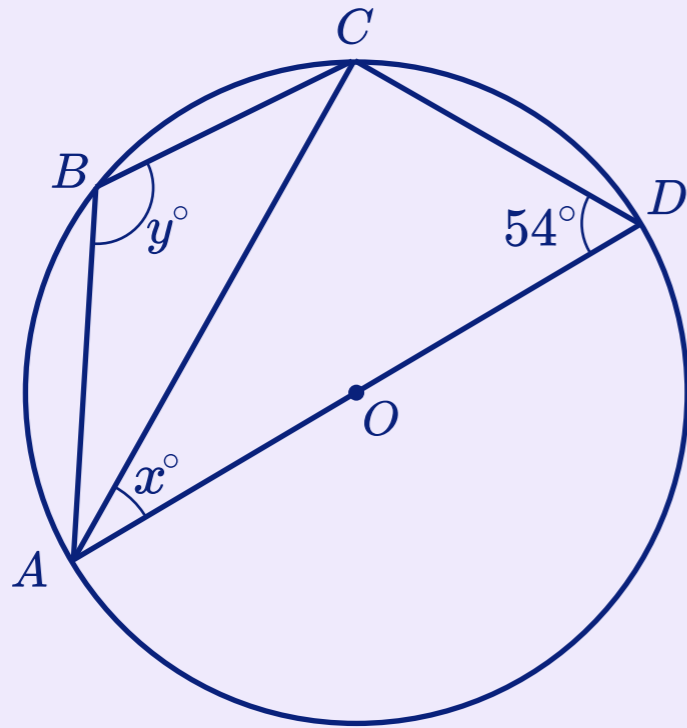
$$y = \dots\dots\dots$$

Reason:

.....  
 .....

## Your turn...

The points  $A$ ,  $B$ ,  $C$  and  $D$  are on the circumference of a circle centre  $O$ .



a) Find the value of  $x$ .

Give reasons for your answer.

(3)

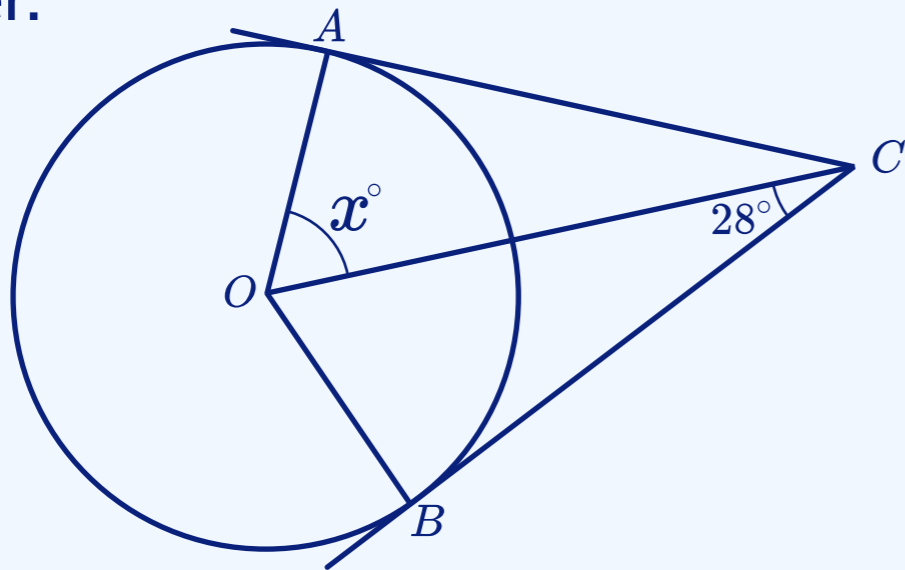
b) Find the value of  $y$ .

Give reasons for your answer.

(2)

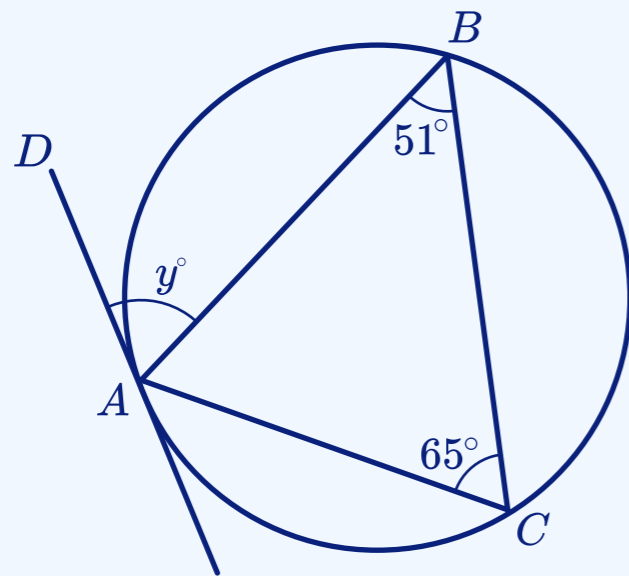
Try this exam style question...

- a)  $AC$  and  $BC$  are tangents to the circle.  
Find the value of  $x$ . Give reasons for your answer.



(3)

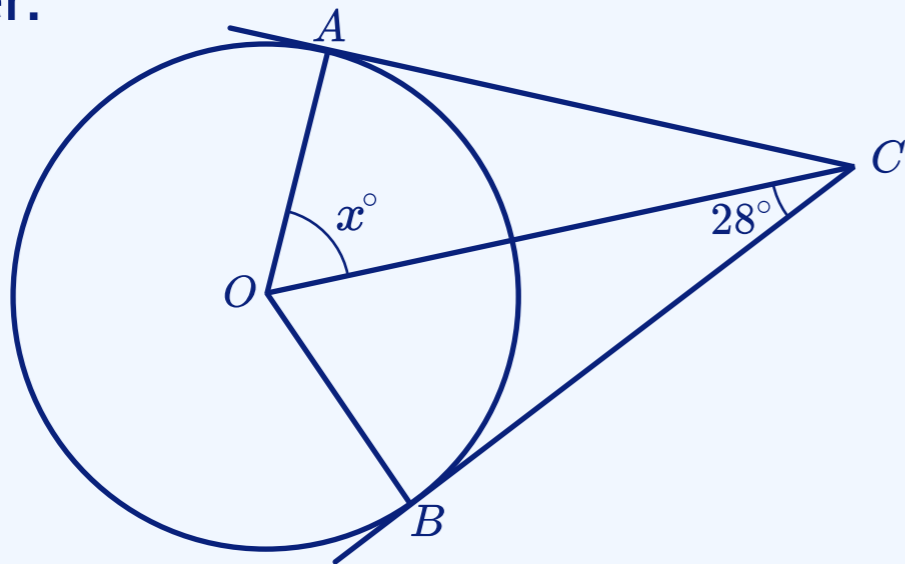
- b)  $AD$  is a tangent to the circle.  
Find the value of  $y$ . Give reasons for your answer.



(2)

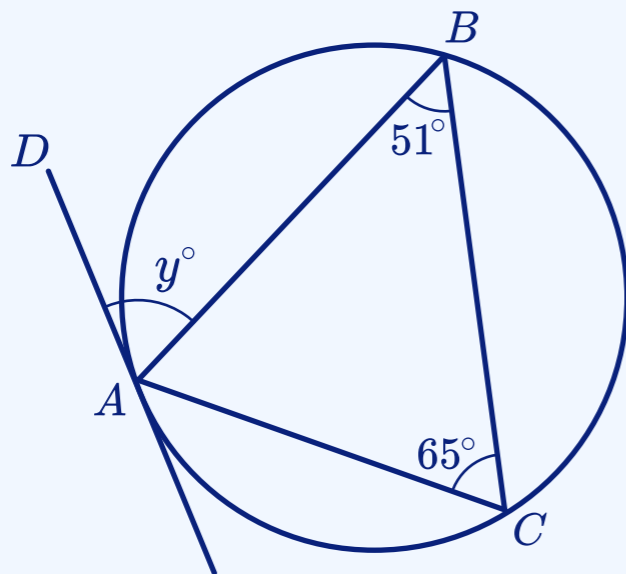
## Let's go through it together...

a)  $AC$  and  $BC$  are tangents to the circle. Find the value of  $x$ . Give reasons for your answer.



(3)

b)  $AD$  is a tangent to the circle. Find the value of  $y$ . Give reasons for your answer.



(2)

**Tangents** are lines that touch a curve or circle at one point.

Here are the circle theorems involving tangents.

### Tangent Of A Circle

Diagram A

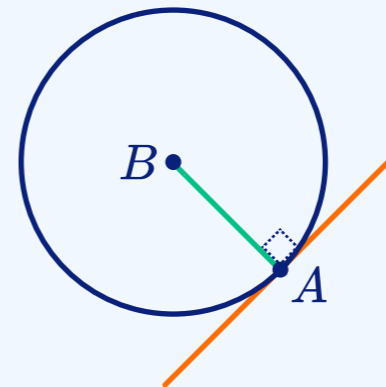
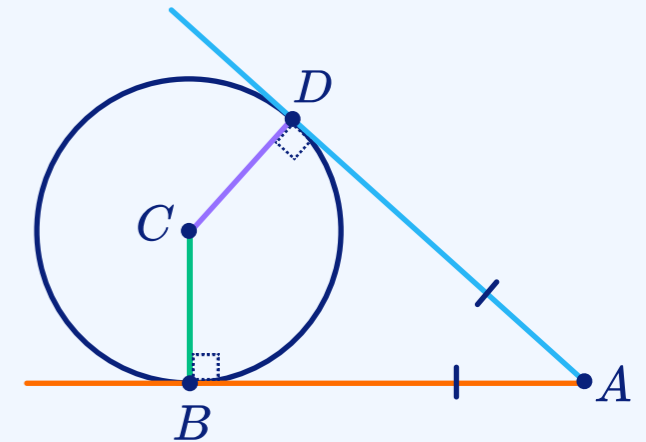


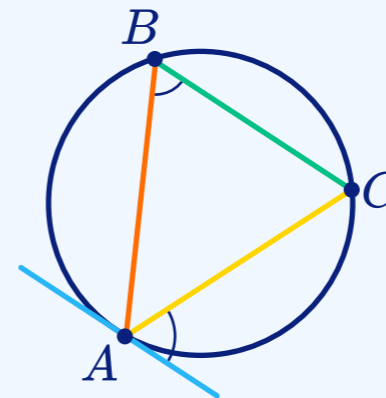
Diagram B



A. The angle between a tangent and radius is 90 degrees.

B. Tangents which meet at the same point are equal in length.

### Alternate Segment Theorem



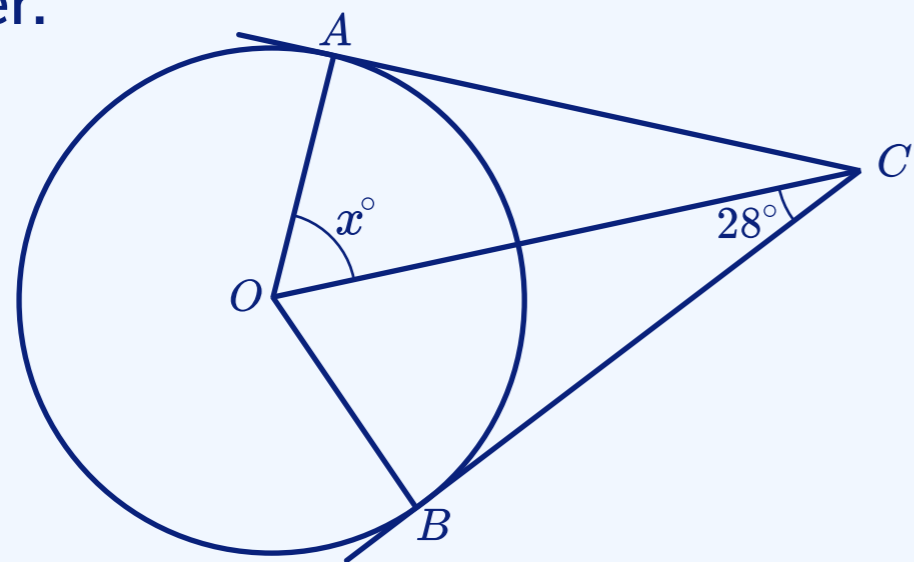
The angle that lies between a tangent and a chord is equal to the angle subtended by the same chord in the alternate segment.

For this circle theorem we can just state "Alternate Segment Theorem" when giving a reason for an answer.

1 Can you match the diagrams in the questions to the circle theorem?

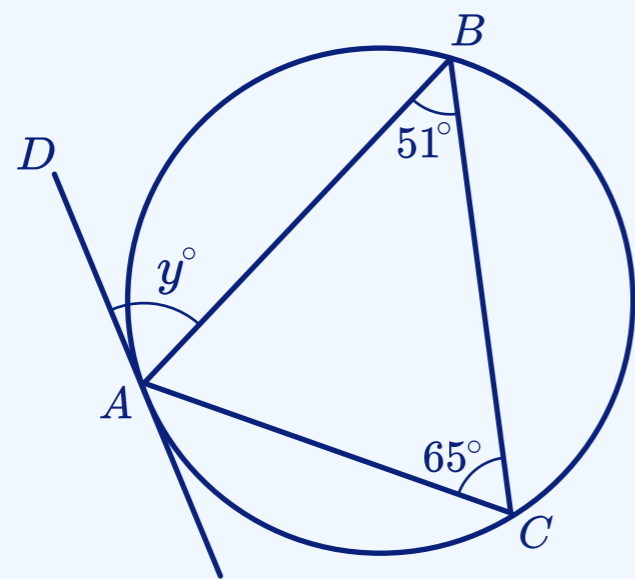
# Let's go through it together...

a)  $AC$  and  $BC$  are tangents to the circle. Find the value of  $x$ . Give reasons for your answer.



(3)

b)  $AD$  is a tangent to the circle. Find the value of  $y$ . Give reasons for your answer.



(2)

2 Find the size of  $\angle OAC$  and state the reason.

$$\angle OAC = \dots\dots\dots$$

Reason:   
 .....  
 .....

3  $AC$  and  $BC$  are the same length, so which angle is the same size as  $\angle OCB$ ?

.....

4 Find the size of  $x$  and state the reason.

$$x = \dots\dots\dots$$

Reason:   
 .....  
 .....

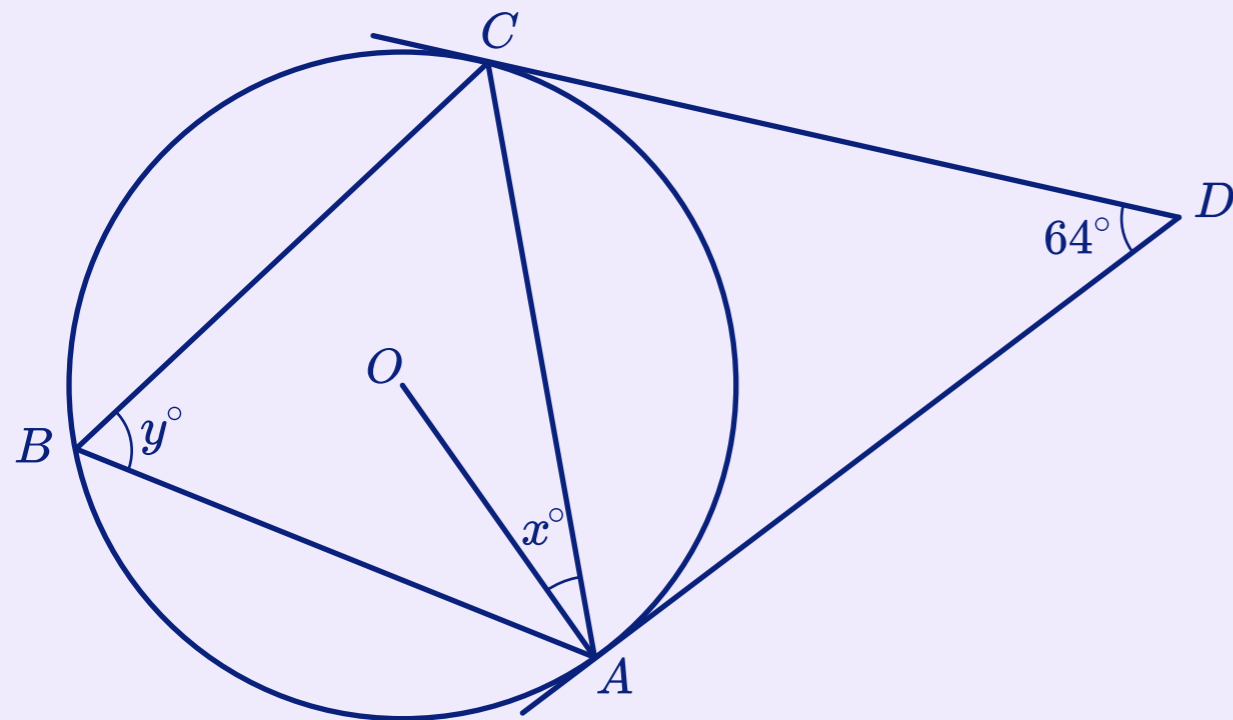
5 Find the size of  $y$  and state the reason.

$$y = \dots\dots\dots$$

Reason:   
 .....  
 .....

## Your turn...

$AD$  and  $CD$  are tangents to the circle.



a) Find the value of  $x$ .

Give reasons for your answer.

(4)

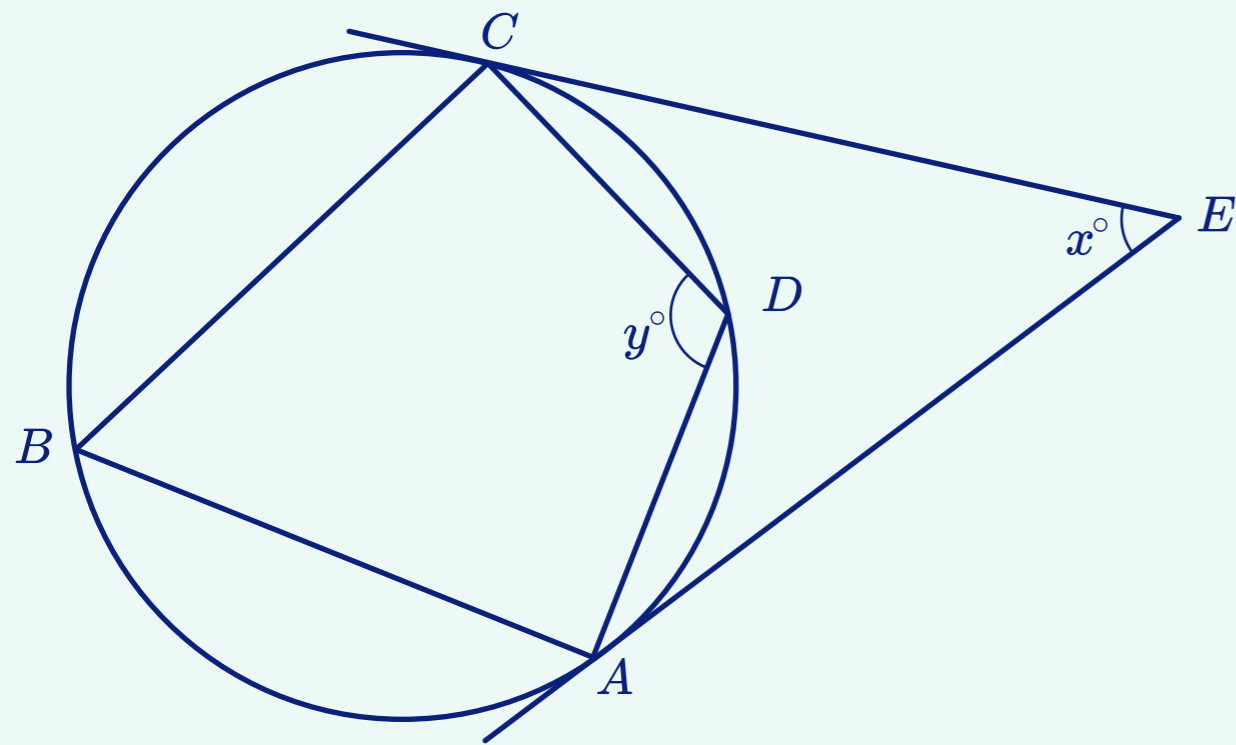
b) Find the value of  $y$ .

Give reasons for your answer.

(2)

## Ready for a Challenge?

$AE$  and  $CE$  are tangents to the circle.



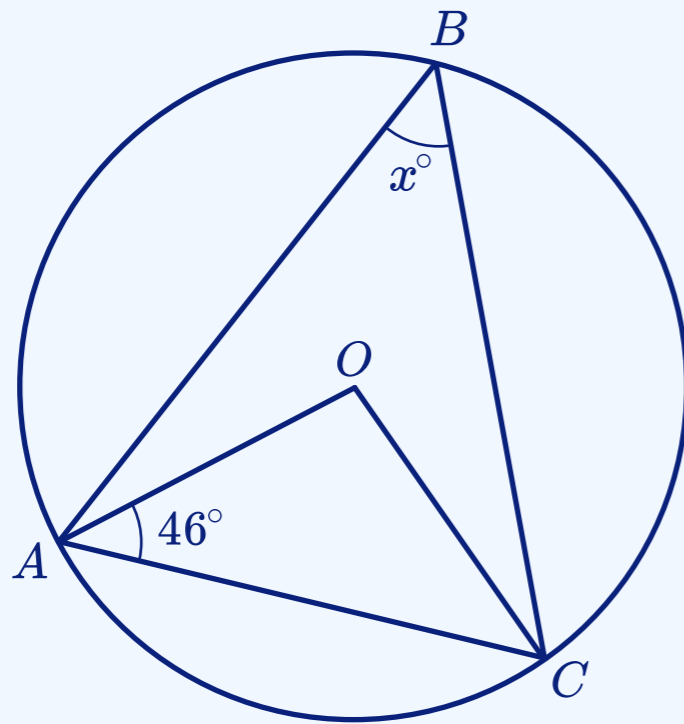
Write an expression for  $y$  in term of  $x$ .

Sometimes it may help to **draw extra lines** on a circle diagram to uncover helpful circle theorems.

## What have we learnt?

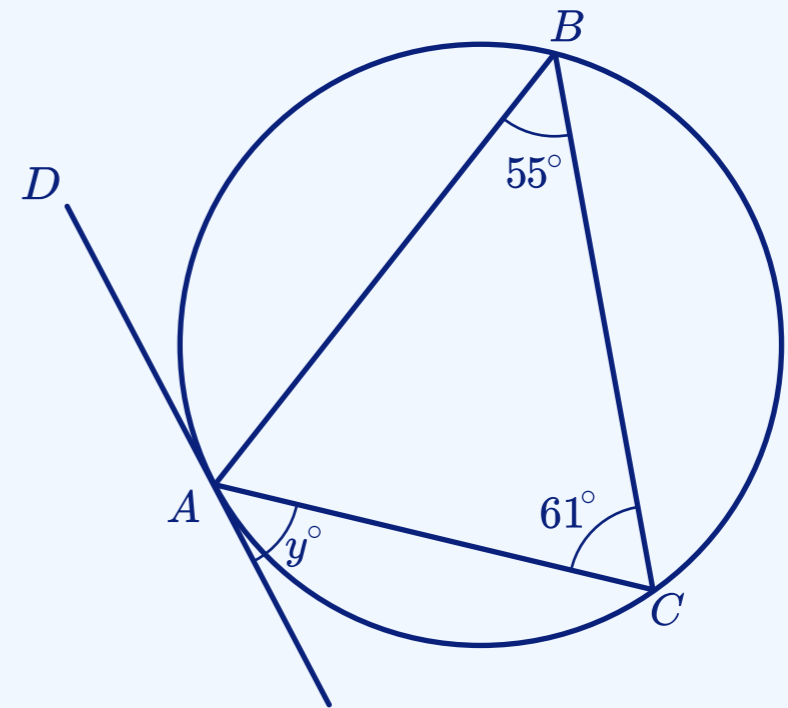
Can you correct the answers to the questions below?

Find the value of  $x$ .



.....  
 $46^\circ$

$AD$  is the tangent to the circle.  
Find the value of  $y$ .



.....  
 $61^\circ$

# Where to go next?

For more diagnostic questions, and GCSE maths revision resources and worksheets to support students in fixing any misconceptions take a look at the free Third Space Learning [GCSE maths revision](#) pages.

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