



THIRD SPACE
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GCSE Maths Intervention Pack

Finding the n^{th} Term of a
Quadratic Sequence

Grade 7

Teacher Notes

Question Sets

Set 1: Finding n^{th} terms in the form $n^2 + c$

Calculate the n^{th} term of a quadratic sequence of the form $n^2 + c$

Key words: Arithmetic sequence, coefficient, common 2nd difference, constant, expression, 1st difference, quadratic sequence, square numbers, term

Set 2: Finding n^{th} terms in the form $n^2 + bn + c$

Calculate the n^{th} term of a quadratic sequence of the form $n^2 + bn + c$

Key words: Arithmetic sequence, coefficient, common 2nd difference, constant, expression, 1st difference, quadratic sequence, square numbers, term

Set 3: Finding n^{th} terms in the form $an^2 + bn + c$

Calculate the n^{th} term of a quadratic sequence of the form $an^2 + bn + c$

Key words: Arithmetic sequence, coefficient, common 2nd difference, constant, expression, 1st difference, quadratic sequence, square numbers, term



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“We now tell our staff that if Third Space Learning do a version of that resource, prioritise it over all of the alternatives, because we feel that they are always the best ones.”



Gabriel Ogbeifun,
Head of Mathematics, Regent High School

Slide 1: Cover Slide

Teaching Prompts

- Can you find the difference between 4 and 9? 9 and 16?
 - Now find the differences between that row.
 - Can you see the connection between the 2nd difference and the coefficient of n^2 (the coefficient is half the 2nd difference)
-

Answers

n^2	$2n^2$	$3n^2$
1, 4, 9, 16,...	2, 8, 18, 32,...	3, 12, 27, 48,...
$\begin{array}{c} \text{+3} \quad \text{+5} \quad \text{+7} \\ \hline \text{+2} \quad \text{+2} \end{array}$	$\begin{array}{c} \text{+6} \quad \text{+10} \quad \text{+14} \\ \hline \text{+4} \quad \text{+4} \end{array}$	$\begin{array}{c} \text{+9} \quad \text{+15} \quad \text{+21} \\ \hline \text{+6} \quad \text{+6} \end{array}$

What do you notice? **The second difference is double the coefficient of n^2 .**

Teacher Reference Only

Common Misconceptions

- Students can fail to recognise quadratic sequences and can assume they are arithmetic (linear) or geometric sequences.
- Students can forget the half the 2nd difference when trying to find the n^{th} term.
- Students may be able to find the correct quadratic coefficient but then be unable to find the remaining arithmetic sequence n^{th} term.

Slide 2: Try this exam-style question...

Set 1: Finding n^{th} terms in the form $n^2 + c$.

Teaching Prompts

- Can you try this question by yourself?
-

If Stuck

- Move on to the next slide.
-

Mark Scheme

- (1 mark) for finding 2nd difference of 2
- (1 mark) n^2+5

Slide 3: Let's go through it together...

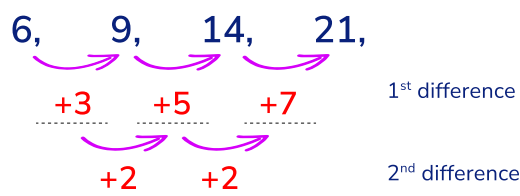
Set 1: Finding n^{th} terms in the form $an^2 + b$

Teaching Prompts

- Can you find the 1st difference and 2nd difference of the sequence?
- What must we do to the 2nd difference to find the multiple of n^2 ?
- What do we get when we subtract the quadratic part of the sequence?

Answers

1. Find the 1st and 2nd differences.



2. Half the 2nd difference to find the multiple of n^2 and subtract it from the original sequence.

$$\begin{array}{r}
 6, \quad 9, \quad 14, \quad 21, \\
 - \quad 1 \quad 4 \quad 9 \quad 16 \\
 \hline
 5 \quad 5 \quad 5 \quad 5
 \end{array}$$

3. Use the constant or arithmetic sequence to write the n^{th} term of the quadratic sequence.

$$n^2 + 5$$

Mark Scheme

- (1 mark) for finding 2nd difference of 2
- (1 mark) $n^2 + 5$

Slide 4: Try this exam-style question

Set 1: Finding n^{th} terms in the form $an^2 + b$

Teaching Prompts

- Can you try this question by yourself?
-

If Stuck

- Can you find the 1st difference and 2nd difference of the sequence?
 - What must we do to the 2nd difference to find the multiple of n^2 ?
 - What do we get when we subtract the quadratic part of the sequence?
-

Mark Scheme

- (1 mark) for finding 2nd difference of 2
- (1 mark) $n^2 - 2$

Slide 5: Try this exam-style question...

Set 2: Finding n^{th} terms in the form $n^2 + bn + c$.

Teaching Prompts

- Can you try this question by yourself?
-

If Stuck

- Move on to the next slide.
-

Mark Scheme

- (1 mark) for finding 2nd difference of 2
- (1 mark) For writing a sequence in the form $n^2 + bn + c$
- (1 mark) $n^2 + 3n - 1$

Slide 6: Let's go through it together...

Set 2: Finding n^{th} terms in the form $n^2 + bn + c$.

Teaching Prompts

- Can you find the 1st difference and 2nd difference of the sequence?
- What must we do to the 2nd difference to find the coefficient of n^2 ?
- What do we get when we subtract the quadratic part of the sequence?
(an arithmetic sequence)

Answers

1. Find the 1st and 2nd differences.

$$\begin{array}{ccccccc}
 3, & 9, & 17, & 27, & & & \\
 \swarrow & \searrow & \swarrow & \searrow & & & \\
 +6 & +8 & +10 & & & & \\
 \hline
 & \swarrow & \searrow & & & & \\
 & +2 & +2 & & & & \\
 \hline
 & & & & & &
 \end{array}$$

1st difference

2nd difference

2. Half the 2nd difference to find the multiple of n^2 and subtract it from the original sequence.

$$\begin{array}{ccccccc}
 3, & 9, & 17, & 27, & & & \\
 - & 1 & 4 & 9 & 16 & & \\
 \hline
 2 & 5 & 8 & 11 & & &
 \end{array}$$

$3n - 1$
arithmetic n^{th} term

3. Use the constant or arithmetic sequence to write the n^{th} term of the quadratic sequence.

$$n^2 + 3n - 1$$

Mark Scheme

- (1 mark) for finding 2nd difference of 2
- (1 mark) For writing a sequence in the form $n^2 + bn + c$
- (1 mark) $n^2 + 3n - 1$

Slide 7: Your turn...

Set 2: Finding n^{th} terms in the form $n^2 + bn + c$.

Teaching Prompts

- Can you find the 1st difference and 2nd difference of the sequence?
 - What must we do to the 2nd difference to find the coefficient of n^2 ?
 - What do we get when we subtract the quadratic part of the sequence?
(an arithmetic sequence)
-

Mark Scheme

- (1 mark) for finding 2nd difference of 2
- (1 mark) For writing a sequence in the form $n^2 + bn + c$
- (1 mark) $n^2 + 5n + 1$

Slide 8: Try this exam-style question...

Set 3: Finding n^{th} terms in the form $an^2 + bn + c$.

Teaching Prompts

- Can you try this question by yourself?
-

If Stuck

- Move on to the next slide.
-

Mark Scheme

- (1 mark) For finding 2nd difference of 6
- (1 mark) For writing a sequence in the form $3n^2 + bn + c$
- (1 mark) $3n^2 - 2n + 10$

Slide 9: Let's go through it together...

Set 3: Finding n^{th} terms in the form $an^2 + bn + c$.

Teaching Prompts

- Can you find the 1st difference and 2nd difference of the sequence?
- What will be the multiple of n^2 ? (3)
- Can you write the first 4 terms of the multiple of n^2 ? (3, 12, 27, 48)
- What do we get when we subtract the quadratic part of the sequence?
(an arithmetic sequence)

Answers

1. Find the 1st and 2nd differences.

$$\begin{array}{ccccccc}
 11, & 18, & 31, & 50, & \dots & & \\
 \text{---} & \text{---} & \text{---} & \text{---} & & & \\
 & +7 & +13 & +19 & & & \text{1st difference} \\
 \text{---} & \text{---} & \text{---} & \text{---} & & & \\
 & +6 & +6 & & & & \text{2nd difference} \\
 \text{---} & \text{---} & \text{---} & \text{---} & & &
 \end{array}$$

2. Half the 2nd difference to find the multiple of n^2 and subtract it from the original sequence.

$$\begin{array}{ccccccc}
 11, & 18, & 31, & 50, & & & \\
 - & 3, & 12, & 27, & 48, & 3n^2 & \\
 \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & & \\
 & 8, & 6, & 4, & 2, & & \\
 \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & &
 \end{array}$$

$-2n + 10$
 arithmetic n^{th} term

3. Use the constant or arithmetic sequence to write the n^{th} term of the quadratic sequence.

$$3n^2 - 2n + 10$$

Mark Scheme

- (1 mark) for finding 2nd difference of 6
- (1 mark) For writing a sequence in the form $3n^2 + bn + c$
- (1 mark) $3n^2 + 2n + 10$

Slide 10: Your turn...

Set 3: Finding n^{th} terms in the form an^2+bn+c .

Teaching Prompts

- Can you try this question by yourself?
-

If Stuck

- Can you find the 1st difference and 2nd difference of the sequence?
 - What will be the multiple of n^2 ? (2)
 - Can you write the first 4 terms of the multiple of n^2 ? (2, 8, 18, 32)
 - What do we get when we subtract the quadratic part of the sequence? (an arithmetic sequence)
-

Mark Scheme

- (1 mark) For finding 2nd difference of 4
- (1 mark) For writing a sequence in the form $2n^2+bn+c$
- (1 mark) $2n^2+4n-5$

Slide 11: Ready for a Challenge?

Teaching Prompts

- Can you try this question by yourself?
-

If Stuck

- Can you find the 1st difference and 2nd difference of the sequence?
 - What will be the multiple of n^2 ? (0.5)
 - Can you write the first 4 terms of the multiple of n^2 ? (0.5, 2, 4.5, 8)
 - What do we get when we subtract the quadratic part of the sequence? (an arithmetic sequence)
-

Mark Scheme

- (1 mark) For finding 2nd difference of 1
- (1 mark) For writing a sequence in the form $0.5n^2 + bn + c$
- (1 mark) $0.5n^2 + 0.5n + 3$

Slide 12: What have we learnt?

Teaching Prompts

- Can you see where the student has gone wrong? (they have assumed it was a arithmetic sequence by just looking at the first 2 terms)
- What should they have done instead?
- Found the 1st and 2nd difference to find the n^{th} term of a quadratic sequence.

0, 3, 8, 15, ...

3 5 7
2 2

0 3 8 15
- 1 4 9 16
-1 -1 -1 -1

- Answer = $n^2 - 1$
- Can you see where the student has gone wrong? (they have not halved the 2nd difference)
- What should they have done instead?
- Halved the 2nd difference to get $2n^2$, then subtracted to find the remaining terms

5, 12, 23, 38, ...

- 2 8 18 32 $2n^2$
3 4 5 6 $n + 2$

- Answer = $2n^2 + n + 2$

Finding the n^{th} Term of a Quadratic Sequence

Complete the 1st and 2nd differences for these sequences.
What do you notice?

$$\begin{array}{ccccccc} & & n^2 & & & & \\ & & 1, & 4, & 9, & 16, & \dots \\ & \text{↗} & & \text{↗} & & \text{↗} & \\ & +3 & & & & & \\ & \dots & & \dots & & \dots & \\ & \text{↗} & & \text{↗} & & & \\ & \dots & & \dots & & \dots & \end{array}$$

$$\begin{array}{ccccccc} & & 2n^2 & & & & \\ & & 2, & 8, & 18, & 32, & \dots \\ & \text{↗} & & \text{↗} & & \text{↗} & \\ & +6 & & & & & \\ & \dots & & \dots & & \dots & \\ & \text{↗} & & \text{↗} & & & \\ & \dots & & \dots & & \dots & \end{array}$$

$$\begin{array}{ccccccc} & & 3n^2 & & & & \\ & & 3, & 12, & 27, & 48, & \dots \\ & \text{↗} & & \text{↗} & & \text{↗} & \\ & +9 & & & & & \\ & \dots & & \dots & & \dots & \\ & \text{↗} & & \text{↗} & & & \\ & \dots & & \dots & & \dots & \end{array}$$

Try this exam-style question...

Here are the first four terms of a quadratic sequence 6, 9, 14, 21,...

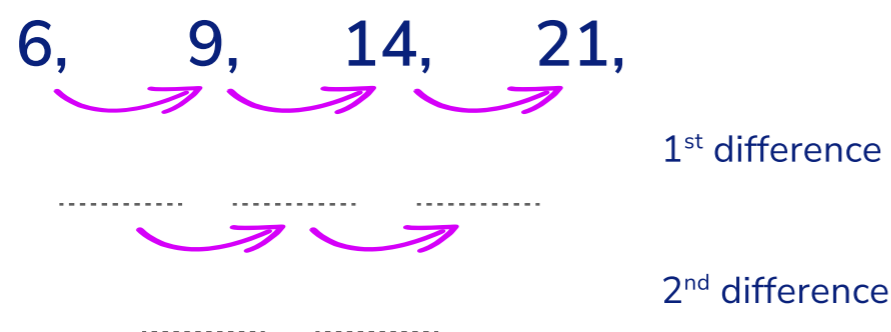
Find an expression for the n th term of this sequence.

Let's go through it together...

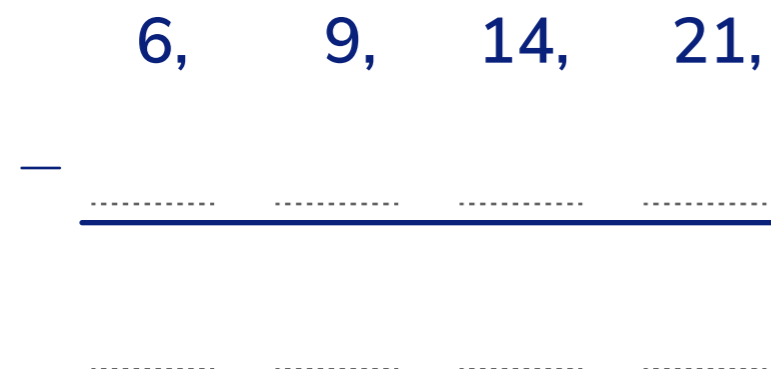
Here are the first four terms of a quadratic sequence 6, 9, 14, 21,...

Find an expression for the n th term of this sequence.

1 Find the 1st and 2nd differences.

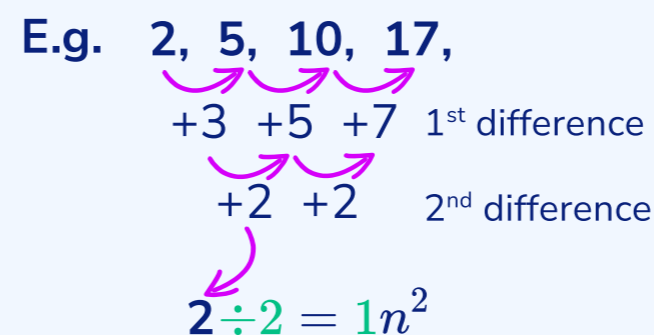


2 Half the 2nd difference to find the multiple of n^2 and subtract it from the original sequence.



3 Use the constant or arithmetic sequence to write the n th term of the quadratic sequence.

A **quadratic sequence** will have a **common 2nd difference**.
The **square numbers** 1, 4, 9, 16, ... , have a common 2nd difference of +2 and an n th term of n^2 .



Halving the 2nd difference gives the multiple of n^2 .

Subtracting that multiple of n^2 from the original sequence gives a **constant**.



To get the n th term we **add the constant** to the multiple of n^2 .

n th term of 2, 5, 10, 17, ..., is $1n^2 + 1$
We can write this as $n^2 + 1$

Here are the first four terms of a quadratic sequence -1, 2, 7, 14,...

Find an expression for the n th term of this sequence.

Try this exam-style question...

Here are the first four terms of a quadratic sequence 3, 9, 17, 27,...

Find an expression for the n th term of this sequence.

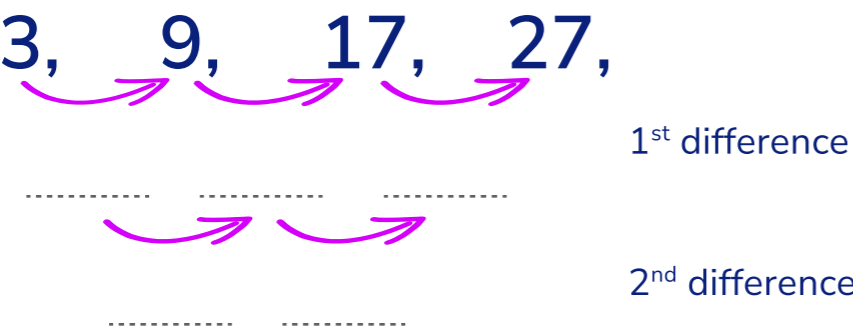
Let's go through it together...

Here are the first four terms of a quadratic sequence

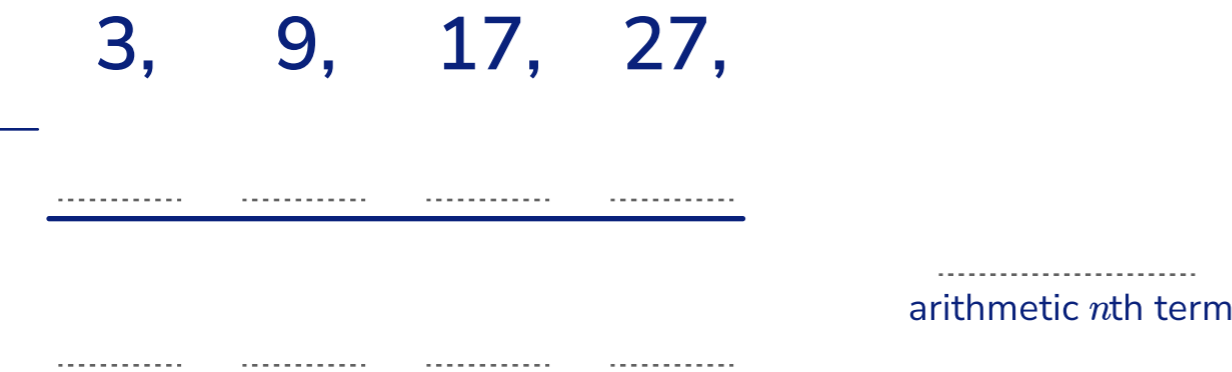
3, 9, 17, 27,...

Find an expression for the n th term of this sequence.

- 1 Find the 1st and 2nd differences.



- 2 Half the 2nd difference to find the multiple of n^2 and subtract it from the original sequence.



- 3 Use the constant or arithmetic sequence to write the n th term of the quadratic sequence.

E.g. 2, 9, 18, 29,

+7 +9 +11 1st difference

+2 +2 2nd difference

÷2 → 1 n^2

2, 9, 18, 29,

— 1, 4, 9, 16, 1 n^2

1, 5, 9, 13, arithmetic sequence

n th term is $4n - 3$

multiple of n^2 + arithmetic sequence n th term

$= n^2 + 4n - 3$

Here are the first four terms of a quadratic sequence 7, 15, 25, 37,...

Find an expression for the n th term of this sequence.

Try this exam-style question...

Here are the first four terms of a quadratic sequence 11, 18, 31, 50,...

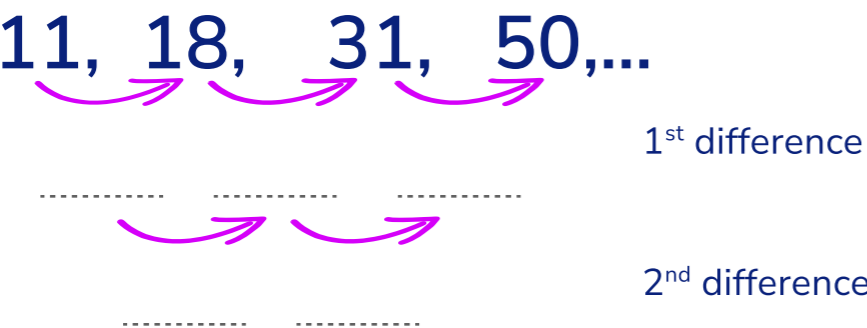
Find an expression for the n th term of this sequence.

Let's go through it together...

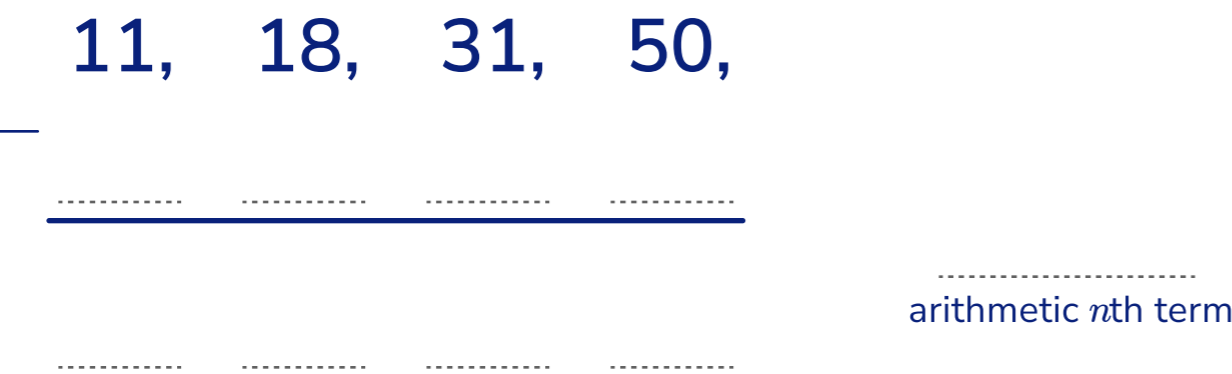
Here are the first four terms of a quadratic sequence 11, 18, 31, 50,...

Find an expression for the n th term of this sequence.

- 1 Find the 1st and 2nd differences.



- 2 Half the 2nd difference to find the multiple of n^2 and subtract it from the original sequence.



- 3 Use the constant or arithmetic sequence to write the n th term of the quadratic sequence.

E.g. 3, 7, 15, 27,

+4 +8 +12 1st difference

+4 +4 2nd difference

÷2 → $2n^2$

3, 7, 15, 27,

— 2, 8, 18, 32, $2n^2$

1, -1, -3, -5, arithmetic sequence
 n th term is $-2n + 3$

multiple of n^2 + arithmetic sequence n th term

$= 2n^2 - 2n + 3$

Here are the first four terms of a quadratic sequence 1, 11, 25, 43,...

Find an expression for the n th term of this sequence.

Ready for a Challenge?

Here are the first four terms of a quadratic sequence 4, 6, 9, 13,...

Find an expression for the n th term of this sequence.

What have we learnt?

Can you correct the answers to the questions below?

Here are the first four terms of a quadratic sequence.

0, 3, 8, 15, ...

Find an expression for the n th term of this sequence.

$$3n - 3$$

Here are the first four terms of a quadratic sequence.

5, 12, 23, 38, ...

Find an expression for the n th term of this sequence.

5 12 23 38
7 11 15
4 4

$$4n^2 + 1$$

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