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GCSE Maths Intervention Pack

Factorising Quadratics (coefficient not equal to 1)

Grade 6

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Teacher Notes

Question Sets

Set 1: Factorising quadratics with prime x^2 coefficients

Factorising quadratics of the form $ax^2 + bx + c = 0$ where a is a prime number

Key words: Coefficient, constant, factorise, factors, prime, quadratic

Set 2: Factorising quadratics with non-prime x^2 coefficients

Factorising quadratics of the form $ax^2 + bx + c = 0$ where a is a not a prime number

Key words: Coefficient, constant, factorise, factors, quadratic

Set 3: Difference of two squares $ax^2 - c$

Factorise quadratic equations into the form

$(ax + b)(ax - b)$ and $(ax + by)(ax - by)$

Key words: Coefficient, constant, difference, factorise, quadratic, square numbers, square root

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Gabriel Ogbeifun,
Head of Mathematics, Regent High School

Slide 1: Cover Slide

Teaching Prompts

- Do you agree with Billy?

Answers

Billy is not correct. The product of the two numbers is equal to $ac = 2 \times -12 = -24$, not $c = -12$. The factor pairs of 24 are: 1×24, 2×12, 3×8 and 4×6. As ac is negative and b is positive, one factor of ac must be negative, and as no combination of these factors can add to get b, the quadratic cannot be factorised.

Teacher Reference Only

Common Misconceptions

- Once students can factorise a quadratic expression of the form $x^2 + bx + c = 0$ they might overcomplicate the simpler case. E.g. $x^2 + 2x = (x + 0)(x + 2)$
- Students may think that $(x + a)^2 = x^2 + a^2$
- Students may forget how to multiply negative numbers.
- Students may think that $x^2 + 12 + 7x$ is not equivalent to $x^2 + 7x + 12$

Terminology

- Factorise: Rewrite the expression or equation by removing a common factor from multiple terms, introducing brackets. E.g. $x^2 + 4x = x\,(x + 4)$
- Quadratic expression: A term or collection of terms where the highest power (usually of x) is 2 e.g $x^2 + 5x + 6$
- Quadratic equation: An equality that contains terms where the highest power (usually of x) is 2 e.g $x^2 + 5x + 6 = 0$

Slide 2: Try this exam-style question...

Set 1: Factorising Quadratics with Prime x^2 Coefficients.

Teaching Prompts

- Can you try this question by yourself?

If Stuck

- Move on to the next slide.

Mark Scheme

- (1 mark) Correctly identifies 2 and 5
- (1 mark) $(2x + 5)(x + 2)$

Watch out for

- Once students can factorise a quadratic expression of the form $x^2 + bx + c = 0$ they might overcomplicate the simpler case. E.g. $x^2 + 2x = (x + 0)(x + 2)$
- Students may think that $(x + a)^2 = x^2 + a^2$
- Students may forget how to multiply negative numbers.
- Students may think that $x^2 + 12 + 7x$ is not equivalent to $x^2 + 7x + 12$

Slide 3: Let's go through it together...

Set 1: Factorising Quadratics with Prime x^2 Coefficients.

Teaching Prompts

- 1) What numbers multiply to make 20?

Answers

1. Find ac and write out its factor pairs.

$ac = 2 \times 10 = 20$

$1 \quad 20$
$2 \quad 10$
$4 \quad 5$

Factorise Columns

$\times$	x	$+2$
$2x$	$2x^2$	$4x$
$+5$	$5x$	$+10$

Factorise Rows

$(2x+5)(x+2)$

2. Circle the factor pair which adds to make b.

$b = 9 = 4 + 5$

Mark Scheme

- (1 mark) Correctly identifies 2 and 5
- (1 mark) $(2x+5)(x+2)$

Slide 4: Your turn...

Set 1: Factorising Quadratics with Prime x^2 Coefficients.

Teaching Prompts

- Can you use a grid to help you?
- What would ac be? (-9)
- What numbers multiply to make -9?
- Which of these add to make 8?

Mark Scheme

- (1 mark) Correctly identifies $+3$ and -1 as factors
- (1 mark) $(3m - 1)(m + 3)$

Slide 5: Try this exam-style question...

Set 2: Factorising Quadratics with Non-Prime x^2 Coefficients.

Teaching Prompts

- Can you try this question by yourself?

If Stuck

- Move on to the next slide.

Mark Scheme

- (1 mark) Correctly identifies -5 and $+1$ as factors
- (1 mark) $(2y - 5)(3y + 1)$

Slide 6: Let's go through it together...

Set 2: Factorising Quadratics with Non-Prime x^2 Coefficients.

Answers

1. Find ac and write out its factor pairs.

$ac = 6 \times -5 = -30$

-6	5
-3	10
-2	15

Factorise Columns

×	$-3y$	$+1$
$2y$	$6y^2$	$+2y$
-30	$-15y$	-5

Factorise Rows

$(2y + 5)(3y + 2)$

2. Circle the factor pair which adds to make b.

$b = -13 = 2 + -15$

Mark Scheme

- (1 mark) Correctly identifies -5 and $+1$ as factors
- (1 mark) $(2y - 5)(3y + 1)$

Slide 7: Your turn...

Set 2: Factorising Quadratics with Non-Prime x^2 Coefficients.

Teaching Prompts

- Can you use a grid to help you?
- What would ac be? (-60)
- What numbers multiply to make -60?
- Which of these add to make -4?

Mark Scheme

- (1 mark) Correctly identifies -5 and $+3$ as factors
- (1 mark) $(2a + 3)(2a - 5)$

Slide 8: Try this exam-style question...

Set 3: Using the Difference of Two Squares $ax^2 - c$.

Teaching Prompts

- Can you try this question by yourself?

If Stuck

- Move on to the next slide.

Mark Scheme

- (1 mark) Identifies one correct bracket e.g. $(2a + 3)$ or $(2a - 3)$
- (1 mark) Fully factorises to $(2a + 3)(2a - 3)$

Slide 9: Let's go through it together...

Set 3: Using the Difference of Two Squares $ax^2 - c$.

Teaching Prompts

- What is the square root of each term you can see? (4x and 3)

Answers

1. $\sqrt{4} = 2, \sqrt{9} = 3$
2. $(2a + 3)(2a - 3)$

Mark Scheme

- (1 mark) Identifies one correct bracket e.g. $(2a + 3)$ or $(2a - 3)$
- (1 mark) Fully factorises to $(2a + 3)(2a - 3)$

Slide 10: Your turn...

Set 3: Using the Difference of Two Squares $ax^2 - c$.

Teaching Prompts

- What do you need to do to values a and c? (square root them)

Mark Scheme

- (1 mark) Identifies one correct bracket e.g. $(5y + 3)$ or $(5y - 3)$
- (1 mark) Fully factorises to $(5y + 3)(5y - 3)$

Slide 11: Ready for a Challenge?

Teaching Prompts

- What are the values of a and c? ($a = -6$ and $c = 5$)
- Can you use a grid to help?

Answers

×	1	$+2x$
5	5	$+10x$
$-3x$	$-3x$	$-6x^2$

Mark Scheme

- (1 mark) Identifies one correct bracket e.g. $(5 - 3x)$ or $(1 + 2x)$
- (1 mark) Fully factorises to $(5 - 3x)(1 + 2x)$

Slide 12: What have we learnt?

Teaching Prompts

- Can you see where the student has gone wrong? (they have not completed the table and they have not accounted for the coefficient on x^2)
- What should they have done instead?

$\times$	$3x$	-2
x	x^2	$-2x$
$+4$	$+12x$	-8

- Answer = $(3x-2)(x+4)$

- Can you see where the student has gone wrong? (they have used $+3$ in both brackets when it should be $+3$ and -3)
- What should they have done instead? $(2x+3)(2x-3)$
- Answer = $(2x+3)(2x-3)$

Factorising Quadratics (Coefficient Not Equal to 1)

Do you agree?

Can the expression be factorised?

$$2x^2 + 11x - 12$$

Billy says:

"Yes. We need to find two numbers that add to make 11 and multiply to make -12. The numbers -1 and 12 work, so our answer must be $(2x - 1)(x + 12)$ *"*

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Try this exam-style question...

Factorise

$$2x^2 + 9x + 10$$

.....

(2)

Let's go through it together...

Factorise

$$2x^2 + 9x + 10$$

For any quadratic $ax^2 + bx + c$ we need to find the two numbers which add to make b and multiply to make ac.

e.g. $2x^2 + 7x + 3$

$$ac = 2 \times 3 = 6$$

$1, 6$

$2, 3$

$$b = 7 = 1 + 6$$

This gives us the values we need to **complete the grid below.**

We can then **factorise each column and row** to find each of our brackets!

×	$2x$	$+1$
x	$2x^2$	$+x$
$+3$	$+6x$	$+3$

$$(2x + 1)(x + 3)$$

1 Find ac and write out its factor pairs.

$ac =$ ..... × ..... = ...

....

....

....

2 Circle the factor pair which adds to make b.

$b =$ ..... = ... + ...

Factorise Columns

Factorise Rows

×		
	$2x^2$	☐x
	☐x	$+ 10$

( )( )

(2)

Your turn...

Factorise

$$3m^2 + 8m - 3$$

(2)

Try this exam-style question...

Factorise

$$6y^2 - 13y - 5$$

.....

(2)

Factorising Quadratics with Non-Prime x^2 Coefficients: Exam Q2

Let's go through it together...

Factorise

$$6y^2 - 13y - 5$$

This question is a little more complicated. However, we follow the same process—for a quadratic $ax^2 + bx + c$ we need to find the two numbers which **add** to make b and **multiply** to make ac.

This gives us the values we need to **complete the grid**.

We can then **factorise each column and row** to find each of our brackets!

1 Find ac and write out its factor pairs:

$ac =$ ..... $\times$ ..... $=$ .....

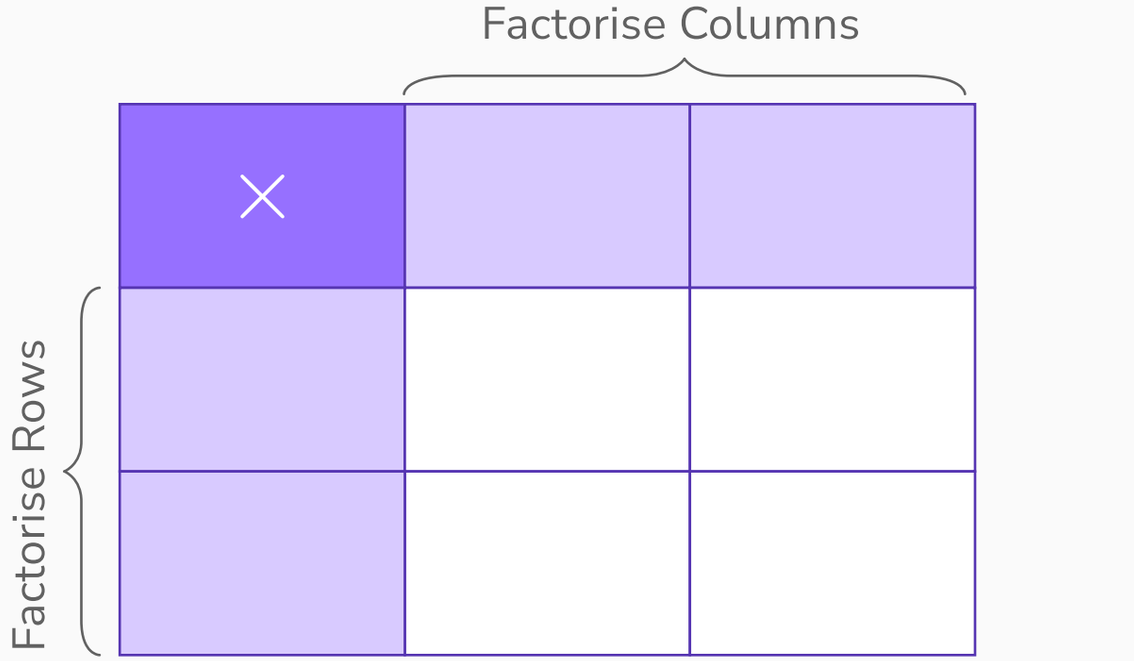
....

....

....

2 Circle the factor pair which adds to make b

$b =$ ..... $=$ ..... $+$ .....



( )( )

(2)

Your turn...

Factorise

$$4a^2 - 4a - 15$$

(2)

Try this exam-style question...

Factorise

$$4a^2 - 9$$

.....

(2)

Difference of two squares ax^2-c: Exam Q3

Let's go through it together...

Factorise

$$4a^2 - 9$$

To factorise a quadratic in the form $ax^2 - c$, where "a" and "c" are both **square numbers**, we simply need to find the **square root of "a" and "c"**.

1 Find the square root of a and c:

$\sqrt{} =$

$\sqrt{} =$

When "a" and "c" are both square numbers:

$$ax^2 - c = (\sqrt{a}x + \sqrt{c})(\sqrt{a}x - \sqrt{c})$$

2 Complete the brackets:

( )( )

(2)

Your turn...

Factorise

$$25y^2 - 9$$

(2)

Ready for a Challenge?

Factorise

$$5 + 7x - 6x^2$$

(2)

What have we learnt?

Can you correct the answers to the questions below?

Factorise

$$3x^2 + 10x - 8$$

$\times$		
	$3x^2$	
		-8

$(3x + 8)(x - 1)$

Factorise

$$4x^2 - 9$$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$(2x + 3)(2x + 3)$

Factorising Quadratics (Coefficient Not Equal to 1): Review

Where to go next?

For more diagnostic questions, and GCSE maths revision resources and worksheets to support students in fixing any misconceptions take a look at the free Third Space Learning GCSE maths revision pages.

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