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# GCSE Maths Intervention Pack

## Solving Linear Equations

Grade 3

## Teacher Notes

### Question Sets

#### Set 1: Solving One Step Equations

Solve one step equations of the form:  $ax = b$ ,  $\frac{x}{a} = b$ , and  $x \pm a = b$ .

Key words: Equation, evaluate, function, inverse, solve, variable (unknown).

#### Set 2: Solving Two Step Equations

Solve two step equations of the form  $ax \pm b = c$ ,  $\frac{x}{a} \pm b = c$ ,  $a(x \pm b) = c$ ,  
and  $\frac{x \pm b}{a} = c$ .

Key words: Equation, evaluate, function, inverse, solve, variable (unknown).

#### Set 3: Solving Equations with 'x' on Both Sides

Solve multi-step equations of the form  $ax \pm b = cx \pm d$  and  $b \pm ax = d \pm cx$ .

Key words: Equation, evaluate, function, inverse, negative coefficient, solve, variable (unknown).



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"We now tell our staff that if Third Space Learning do a version of that resource, prioritise it over all of the alternatives, because we feel that they are always the best ones."



**Gabriel Ogbeifun,**  
Head of Mathematics, Regent High School



## Slide 1: Cover Slide

### Teaching Prompts

- Who do you agree with? Why?
- 

### Answers

- Hafisa is correct because she subtracted 4 first and then divided by 2.
- 

### Teacher Reference Only

#### Common Misconceptions

- Some pupils may think that equations always need to be presented in the form  $ax + b = c$  rather than  $c = ax + b$ . Show examples where this is not the case.
- Some pupils may think that the solution to an equation is always positive and/or a whole number. Show examples where this is not the case.
- Some pupils may use the inverse operations in the wrong order, for example, to solve  $2x + 18 = 38$ , the pupils divide by 2 first and then subtract 18.
- Highlight the importance of order (what has been done to  $x$  and how can we undo it?).
- Some pupils may also apply inverse operations wrong - so for  $2x + 18 = 38$  they might add 18 on the right hand side instead of subtracting.
- Highlight that we always have to apply the same operation on both sides (to keep the equation balanced).
- Some pupils may ensure that the variable stays on the left hand side of the equals sign, which can introduce a negative term. This is not incorrect but students may then get an incorrect solution because of their knowledge of multiplying negative numbers. E.g.  $x = 2x - 4$  would become  $-x = -4$ . A solution of  $x = -4$  is incorrectly given due to misconceptions with negative numbers.
- Some pupils may collect like terms rather than carrying out an inverse. E.g.  $x = 2x - 4$  would become  $3x = -4$ . Here they have added  $2x$  instead of subtracting.

## Slide 2: Try these exam-style questions...

### Set 1: Solving One Step Equations.

#### Teaching Prompts

- Can you answer these questions by yourself?
- 

#### If stuck

- Move on to the next slide.
- 

#### Mark Scheme

- a) (1 mark)  $x = 3$
- b) (1 mark)  $a = 2$
- c) (1 mark)  $n = 24$

## Slide 3: Let's go through it together...

Set 1: Solving One Step Equations.

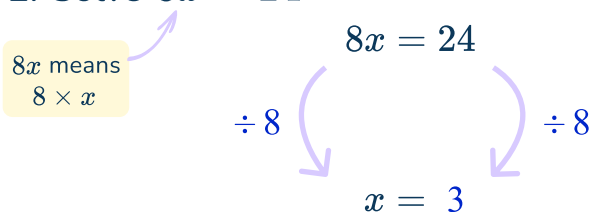
### Teaching Prompts

- a) What is the inverse of multiplying by 8? (dividing by 8)
  - b) What is the inverse of adding 11? (subtracting 11)
- 

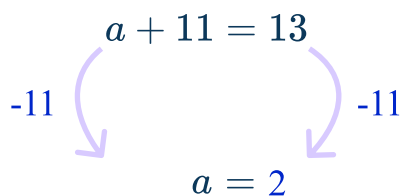
### Answers

1. Dividing, Subtracting

2. Solve  $8x = 24$



3. Solve  $a + 11 = 13$



### Mark Scheme

a) (1 mark)  $x = 3$

b) (1 mark)  $a = 2$

## Slide 4: Let's go through it together...

Set 1: Solving One Step Equations.

### Teaching Prompts

- c) What is the inverse of dividing by 4? (multiplying by 4)
- 

### Answers

4. Multiplying

$$\begin{array}{c} \frac{n}{4} = 6 \\ \swarrow \quad \searrow \\ \times 4 \quad \times 4 \\ n = 24 \end{array}$$

---

### Mark Scheme

c) (1 mark)  $n = 24$

## Slide 5: Your turn...

Set 1: Solving One Step Equations.

### Teaching Prompts

- Can you answer these questions by yourself?
- 

### If stuck

- a) What is the inverse of dividing by 10? (multiplying by 10)
  - b) What is the inverse of subtracting 9? (adding 9)
  - c) What is the inverse of multiplying by 7? (dividing by 7)
- 

### Mark Scheme

- a) (1 mark)  $h = 20$
- b) (1 mark)  $f = 15$
- c) (1 mark)  $e = 6$

## Slide 6: Try these exam-style questions...

### Set 2: Solving Two Step Equations

#### Teaching Prompts

- Can you answer these questions by yourself?
- 

#### If stuck

- Move on to the next slide.
- 

#### Mark Scheme

a) Allow full marks for any other correct method

- (1 mark) adds 8 to both sides
- (1 mark)  $m = 40$

b) Allow full marks for any other correct method

- (1 mark)  $x + 4 = 6$  (or for expanding bracket and then subtracting 12)
  - (1 mark)  $x = 2$
- 

#### Watch out for

- Some pupils may use the inverse operations in the wrong order, for example, to solve  $2x + 18 = 38$  the pupils divide by 2 first and then subtract 18, if the divide by 2 first, they need to divide ALL terms by 2 to get  $x + 9 = 19$ .
- Highlight the importance of order (what has been done to  $x$  and how can we undo it?).
- Some pupils may also apply inverse operations wrong - so for  $2x + 18 = 38$  they might add 18 on the right hand side instead of subtracting.
- Highlight that we always have to apply the same operation on both sides (to keep the equation balanced).

## Slide 7: Let's go through it together...

### Set 2: Solving Two Step Equations

### Teaching Prompts

- a) What is the opposite of subtracting 8? (+8)  
a) What is the opposite of dividing by 5? (-5)
- 

### Answers

1. Undo these steps to find  $m$

$$40 = \times 5 \longleftarrow +8 \longleftarrow 0$$

2. Solve using inverse operations:

$$\begin{array}{ccc}
 \frac{m}{5} - 8 = 0 & & \\
 +8 \swarrow & \frac{m}{5} = 8 & \searrow +8 \\
 \times 5 \swarrow & m = 40 & \searrow \times 5
 \end{array}$$


---

### Mark Scheme

- a) Allow full marks for any other correct method
- (1 mark) adds 8 to both sides
  - (1 mark)  $m = 40$

## Slide 8: Let's go through it together...

### Set 2: Solving Two Step Equations

### Teaching Prompts

- b) What is the opposite of multiplying by 3? (dividing by 3)  
 b) What is the opposite of adding 4? (subtracting 4)
- 

### Answers

$$x \rightarrow + 4 \rightarrow \times 3 = 18$$

1. Undo these steps to find  $x$ .

$$x = -4 \leftarrow \div 3 \leftarrow 18$$

2. Solve using inverse operations:

$$\begin{array}{lcl}
 3(x + 4) = 18 & & \\
 \div 3 \swarrow & & \searrow \div 3 \\
 x + 4 = 6 & & \\
 -4 \swarrow & & \searrow -4 \\
 x = 2 & & 
 \end{array}$$


---

### Mark Scheme

- b) Allow full marks for any other correct method

- (1 mark)  $x + 4 = 6$  (or for expanding bracket and then subtracting 12)
- (1 mark)  $x = 2$



## Slide 9: Your turn...

### Set 2: Solving Two Step Equations

## Teaching Prompts

- Can you answer these questions by yourself?
- 

### If stuck

- a) How can you undo the equation? (subtract 3, divide by 2)
  - b) How can you undo the equation? (multiply by 2, subtract 7)
- 

## Mark Scheme

- a) Allow full marks for any other correct method
  - (1 mark) subtracts 3 from both sides
  - (1 mark)  $f = 9$
- b) Allow full marks for any other correct method
  - (1 mark) multiplies both side by 2
  - (1 mark)  $p = 17$

## Slide 10: Try these exam-style questions...

### Set 2: Solving Equations with 'x' on Both Sides

#### Teaching Prompts

- Can you answer these questions by yourself?
- 

#### If stuck

- Move on to the next slide.
- 

#### Mark Scheme

a) Allow full marks for any other correct method

- (1 mark) subtracts  $3a$  from both sides
- (1 mark) subtracts 10 from both sides
- (1 mark) divides by 2 to get  $a = -6$

b) Allow full marks for any other correct method

- (1 mark) adds  $5a$  to both sides
- (1 mark) subtracts 8 from both sides
- (1 mark) divides by 7 to get  $a = \frac{1}{7}$

c) Allow full marks for any other correct method

- (1 mark) adds  $3g$  to both sides
  - (1 mark) solves to get  $g = 3$
- 

#### Watch out for

- Some pupils may ensure that the variable stays on the left hand side of the equals sign, which can introduce a negative term. This is not incorrect but students may then get an incorrect solution because of their knowledge of multiplying negative numbers. E.g.  $x = 2x - 4$  would become  $-x = -4$ . A solution of  $x = -4$  is incorrectly given due to misconceptions with negative numbers.
- Some pupils may collect like terms rather than carrying out an inverse. E.g.  $x = 2x - 4$  would become  $3x = -4$ . Here they have added  $2x$  instead of subtracting.

## Slide 11: Let's go through it together...

### Set 2: Solving Equations with 'x' on Both Sides

### Teaching Prompts

- a) Which is the smallest variable out of  $5a$  and  $3a$ ? ( $3a$ )
  - a) What is the opposite of adding  $3a$ ? (subtracting  $3a$ )
  - a) How can we undo the equation? (subtract 10)
  - a) How can we undo the equation? (divide by 2)
- 

### Answers

1. Circle the smallest variable out of the following pairs.

$5a$  and  $3a$

2. Solve the equations, by subtracting the smallest variable first.

$$5a + 10 = 3a - 2$$

$$2a + 10 = -2$$

$$2a = 8$$

$$a = 4$$

---

### Mark Scheme

- a) Allow full marks for any other correct method
  - (1 mark) subtracts  $3a$  from both sides
  - (1 mark) subtracts 10 from both sides
  - (1 mark) divides by 2 to get  $a = -6$

## Slide 12: Let's go through it together...

Set 3: Solving Equations With 'x' On Both Sides.

### Teaching Prompts

- b) Which is the smallest variable out of  $-5a$  and  $2a$ ? ( $-5a$ )
- b) What is the opposite of subtracting  $5a$ ? (adding  $5a$ )
- b) How can we undo the equation? (subtract 8) (divide by 7)
- c) Which is the smallest variable out of  $g$  and  $-3g$ ? ( $-3g$ )
- c) What is the opposite of subtracting  $3g$ ? (adding  $3g$ )
- c) How can we undo the equation? (divide by 4)

### Answers

b)

$$\begin{array}{ccc}
 & 2a + 8 = 9 - 5a & \\
 +5a \swarrow & & \searrow +5a \\
 7a + 8 = 9 & & \\
 -8 \swarrow & & \searrow -8 \\
 7a = 1 & & \\
 \div 7 \swarrow & & \searrow \div 7 \\
 a = \frac{1}{7} & & 
 \end{array}$$

c)

$$\begin{array}{ccc}
 & g = 12 - 3g & \\
 +3g \swarrow & & \searrow +3g \\
 4g = 12 & & \\
 \div 4 \swarrow & & \searrow \div 4 \\
 g = 3 & & 
 \end{array}$$

### Mark Scheme

b) Allow full marks for any other correct method

- (1 mark) adds  $5a$  to both sides
- (1 mark) subtracts 8 from both sides
- (1 mark) divides by 7 to get  $a = \frac{1}{7}$

c) Allow full marks for any other correct method

- (1 mark) adds  $3g$  to both sides
- (1 mark) solves to get  $g = 3$

## Slide 13: Your turn...

Set 3: Solving Equations With ' $x$ ' On Both Sides.

### Teaching Prompts

- Can you answer these questions by yourself?
- 

### If stuck

- a) Which is the smallest variable out of  $5b$  and  $7b$ ? ( $7b$ )
  - b) Which is the smallest variable out of  $-4c$  and  $-8c$ ? ( $-8c$ )
  - c) Which is the smallest variable out of  $2c$  and  $-4c$ ? ( $-4c$ )
- 

### Mark Scheme

a) Allow full marks for any other correct method

- (1 mark)  $4 = 2b + 8$
- (1 mark)  $2b = -4$
- (1 mark)  $b = -2$

b) Allow full marks for any other correct method

- (1 mark)  $4c + 5 = 7$
- (1 mark)  $4c = 2$
- (1 mark)  $c = \frac{1}{2}$

c) Allow full marks for any other correct method

- (1 mark)  $6c = 30$
- (1 mark)  $c = 5$

## Slide 14: Ready for a Challenge?

### Teaching Prompts

- Can you answer these challenge questions by yourself?
- 

### If stuck

- a) What steps can we use to undo the operations? ( $\times 9, + 7, \div 2$ )
  - b) Which is the smallest coefficient of  $x$ ? ( $-5x$ )
- 

### Mark Scheme

- a) Allow full marks for any other correct method

- (1 mark)  $2x - 7 = 27$
- (1 mark)  $2x = 34$
- (1 mark)  $x = 17$

- b) Allow full marks for any other correct method

- (1 mark)  $13 + 3x = 8$
- (1 mark)  $3x = -5$
- (1 mark)  $x = -\frac{5}{3}$

## Slide 15: What have we learnt?

### Teaching Prompts

- a) Can you see where the student has made a mistake? (they added 3 on one side and subtracted it from the other).
- a) What should they have done instead?
- b) Can you see where the student has made a mistake? (they added 5a to both sides instead of subtracting the smallest coefficient of a, which is 3a).
- b) What should they have done instead?
- 

### Answers

1)

$$\begin{array}{ccc}
 & 3a - 3 = 9 & \\
 +3 \swarrow & & \searrow +3 \\
 & 3a = 12 & \\
 \div 3 \swarrow & & \searrow \div 3 \\
 & a = 4 & 
 \end{array}$$

2)

$$\begin{array}{ccc}
 & 3a - 3 = 5a - 5 & \\
 -3a \swarrow & & \searrow -3a \\
 & -3 = 2a - 5 & \\
 +5 \swarrow & & \searrow +5 \\
 & 2 = 2a & \\
 \div 2 \swarrow & & \searrow \div 2 \\
 & a = 1 & 
 \end{array}$$

# Solving Linear Equations

Who do you think is correct?

Solve  $2x + 4 = 12$

Remi's workings

$$\begin{array}{l} 2x + 4 = 12 \\ \xrightarrow{+4} 2x = 16 \\ \xrightarrow{\div 2} x = 8 \end{array}$$

Amy's workings

$$\begin{array}{l} 2x + 4 = 12 \\ \xrightarrow{\div 2} x + 4 = 6 \\ \xrightarrow{-4} x = 2 \end{array}$$

Hafisa's workings

$$\begin{array}{l} 2x + 4 = 12 \\ \xrightarrow{-4} 2x = 8 \\ \xrightarrow{\div 2} x = 4 \end{array}$$



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Solve these equations.

a)  $8x = 24$

.....  
(1)

b)  $a + 11 = 13$

.....  
(1)

c)  $\frac{n}{4} = 6$

.....  
(1)

Solve these equations.

a)  $8x = 24$

.....  
(1)

b)  $a + 11 = 13$

.....  
(1)

An equation is a mathematical statement, including an “=” sign.

The “=” tells us that each side of the equation has the same value. To solve an equation, we need to **find the value of the ‘unknown,’** represented by a letter.

We can do this using inverse operations, applying the same calculation on both side:

- 1
- What is the inverse of:  
multiplying? .....  
adding? .....

- 2
- Solve  $8x = 24$   

$8x$  means  
 $8 \times x$

$8x = 24$   
.....  
 $x =$  .....

- 3
- Solve  $a + 11 = 13$   
 $a + 11 = 13$   
.....  
 $a =$  .....

Solve these equations.

c)  $\frac{n}{4} = 6$

An equation is a mathematical statement, including an “=” sign.

The “=” tells us that each side of the equation has the same value. To solve an equation, we need to **find the value of the ‘unknown,’** represented by a letter.

4    What is the inverse of dividing? .....

$$\frac{n}{4} = 6$$

.....

$$n = \text{.....}$$

Solve these equations.

a)  $\frac{h}{10} = 2$

.....  
(1)

b)  $f - 9 = 6$

.....  
(1)

c)  $7e = 42$

.....  
(1)

Solve these equations.

a)  $\frac{m}{5} - 8 = 0$

.....  
(2)

b)  $3(x + 4) = 18$

.....  
(2)

## Solve these equations.

$$\text{a) } \frac{m}{5} - 8 = 0$$

Most equations involve **more than one step** to solve.

Here,  $m$  divided by 5, subtract 8 is equal to 0.

$$m \longrightarrow \div 5 \longrightarrow -8 = 0$$

- 1 Undo these steps to find  $m$ .

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \leftarrow \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \leftarrow 0$$

- 2 Solve using inverse operations.

$$\frac{m}{5} - 8 = 0$$

$$\frac{m}{5} = \dots\dots\dots$$

$m =$  .....

Solve these equations.

b)  $3(x + 4) = 18$

Most equations involve **more than one step** to solve.

Here, add 4, multiplied by 3 is equal to 18.

$$x \xrightarrow{+4} \xrightarrow{\times 3} 18$$

1 Undo these steps to find  $m$ .

$$\begin{array}{ccccccc} & & = & & \xleftarrow{\quad} & & \xleftarrow{\quad} 18 \\ \text{.....} & & & & \text{.....} & & \text{.....} \end{array}$$

2 Solve using inverse operations.

$$3(x + 4) = 18$$

$$x + 4 = \text{.....}$$

$$x = \text{.....}$$

Solve these equations.

a)  $2f + 3 = 21$

.....  
(2)

b)  $\frac{p + 7}{2} = 12$

.....  
(2)



## Try these exam-style questions...

Solve these equations.

a)  $5a + 10 = 3a - 2$

.....  
(3)

b)  $2a + 8 = 9 - 5a$

.....  
(3)

c)  $g = 12 - 3g$

.....  
(2)

Solve these equations.

a)  $5a + 10 = 3a - 2$

.....  
(3)

When there is a variable on **both sides** of the equation, start by **subtracting the smallest variable**.

- 1
- Circle the smallest variable out of the following pairs.

$5a$  and  $3a$

- 2
- Solve the equations, by subtracting the smallest variable first.

$5a + 10 = 3a - 2$

.....  
.....  
.....

Solve these equations.

b)  $2a + 8 = 9 - 5a$

.....  
c)  $g = 12 - 3g$  (3)

This method can help us with questions with **negative coefficients**.  
Subtracting a negative is equivalent to adding the positive.  $- - 5a = +5a$

$2a + 8 = 9 - 5a$

Add  $5a$  to both sides.

.....  
 $g = 12 - 3g$

Add  $3g$  to both sides.

Solve these equations.

a)  $5b + 4 = 7b + 8$

.....  
(3)

b)  $5 - 4c = 7 - 8c$

.....  
(3)

c)  $2c = 30 - 4c$

.....  
(2)

Solve these equations.

a)  $\frac{2x - 7}{9} = 3$

---

(3)

b)  $13 - 2x = 8 - 5x$

---

(3)

Can you correct the answers to the questions below?

Solve the equation.

$$3a - 3 = 9$$

$$\begin{array}{l} -3) \quad 3a = 6 \quad (-3 \end{array}$$

$$\begin{array}{l} \div 3) \quad a = 2 \quad (\div 3 \end{array}$$

$$a = 2$$

Solve the equation.

$$3a - 3 = 5a - 5$$

$$\begin{array}{l} +5a) \quad 8a - 3 = -5 \quad (+5a \end{array}$$

$$\begin{array}{l} -3) \quad 8a = -8 \quad (-3 \end{array}$$

$$\begin{array}{l} \div 8) \quad a = -1 \quad (\div 8 \end{array}$$

$$a = -1$$

# Where to go next?

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