

Quadratic Equations

A **quadratic equation** is a quadratic expression that is equal to something. Quadratic equations contain terms up to x^2 – the highest power for a quadratic equation is 2.

They have the general form $ax^2 + bx + c = 0$

Examples

$$x^2 = 5$$

$$x^2 - 2x + 1 = 0$$

$$x^2 - 16 = 0$$

$$2 = 2x^2 + 3x$$

A quadratic equation can have zero, one or two (real) solutions.

Solving Quadratic Equations by Factorising

Solving quadratic equations by factorising allows us to calculate solutions for the equation.



Example

Solve $x^2 - 8x + 15 = 0$

$$(x - 3)(x - 5) = 0$$



$$x - 3 = 0$$

$$x = 3$$



$$x - 5 = 0$$

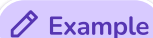
$$x = 5$$

Quadratic Formula

We can solve a quadratic equation using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By using the general form of a quadratic equation $ax^2 + bx + c = 0$ we can substitute the values of a , b and c into the quadratic formula to work out x .



Example Solve $x^2 + 2x - 3 = 0$

$$a = 1 \quad b = 2 \quad c = -3$$

$$x = \frac{-(2) \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-(2) + \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$


$$x = 1$$

$$x = \frac{-(2) - \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$


$$x = -3$$

Completing the Square

If a quadratic expression can be written as a single bracket squared, it is called a **perfect square**.

 **Example** $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$

Most quadratic expressions can't be written like this. **Completing the square** fits the expression to the closest perfect square with an amount added or subtracted.

 **Example** $x^2 + 4x + 7 = (x + 2)^2 + 3$

