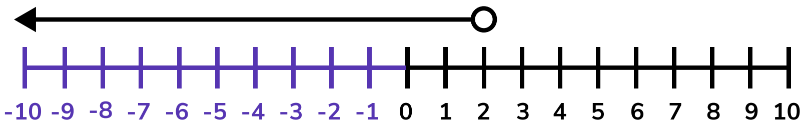
Inequalities

Inequalities compare the size of numbers or expressions.

There are four ways we can compare terms:

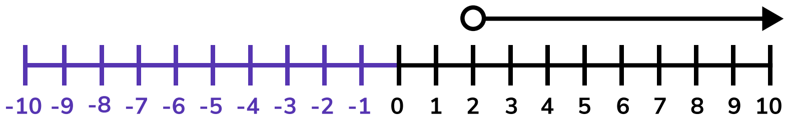
$<$ Less than

Example $x<2$ ‘x is less than 2’



$>$ Greater than

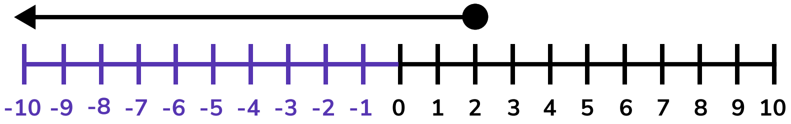
Example $x>2$ ‘x is greater than 2’



$\leq$ Less than or equal to

Example $x\leq 2$

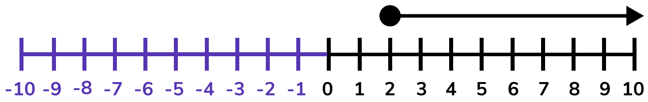
‘x is less than or equal to 2’



$\geq$ Greater than or equal to

Example $x\geq 2$

‘x is greater than or equal to 2’



Solving Inequalities

Solving inequalities is similar to solving equations, but where an equation has one unique solution, an inequality has a range of solutions.

To **solve an inequality** we calculate the values that an unknown variable can be in that inequality.

Example

$$2x + 1 < 9$$
$$\boxed{-1} \quad \boxed{-1}$$
$$2x < 8$$
$$\boxed{\div 2} \quad \boxed{\div 2}$$
$$x < 4$$

Multiplying or dividing by a negative number **changes the direction of the inequality.**

Example

$$1 - 2x < 9$$
$$\boxed{-1} \quad \boxed{-1}$$
$$-2x < 8$$
$$\boxed{\div -2} \quad \boxed{\div -2}$$
$$x > -4$$

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Linear Inequalities

Linear inequalities are inequalities where the power of the unknown in any algebraic expression is no higher than 1.

We can solve linear inequalities in the same way that we solve linear equations.

Example $4x+1<13$ which is read as '$4x+1$ is less than 13'.

$$4x+1<13$$
$$\boxed{-1} \quad \boxed{-1}$$
$$4x<12$$
$$\boxed{\div 4} \quad \boxed{\div 4}$$
$$x<3$$

The solution is $x<3$
This means that x is any value less than (but not including) 3

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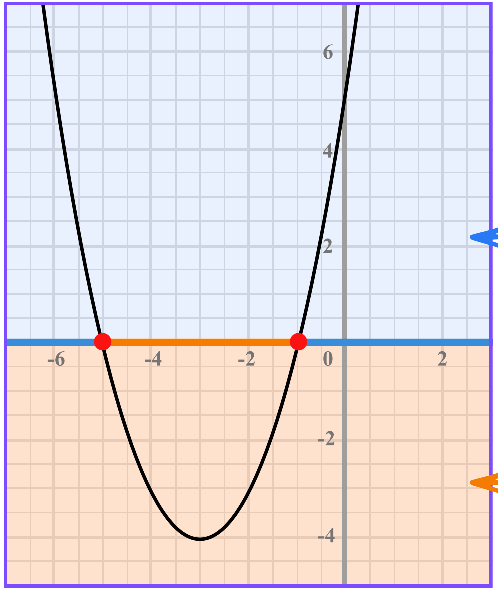
Quadratic Inequalities

A **quadratic inequality** is an inequality where the highest power of any term is 2.

Look for inequality symbols $<, >$ or $\leq, \geq$

Example This is the graph of $y = x^2 + 6x + 5$



We can factorise and solve to find the **roots**:

$$x^2 + 6x + 5 = 0 \Rightarrow (x+1)(x+5) \Rightarrow \begin{matrix} x = -1 \\ x = -5 \end{matrix}$$

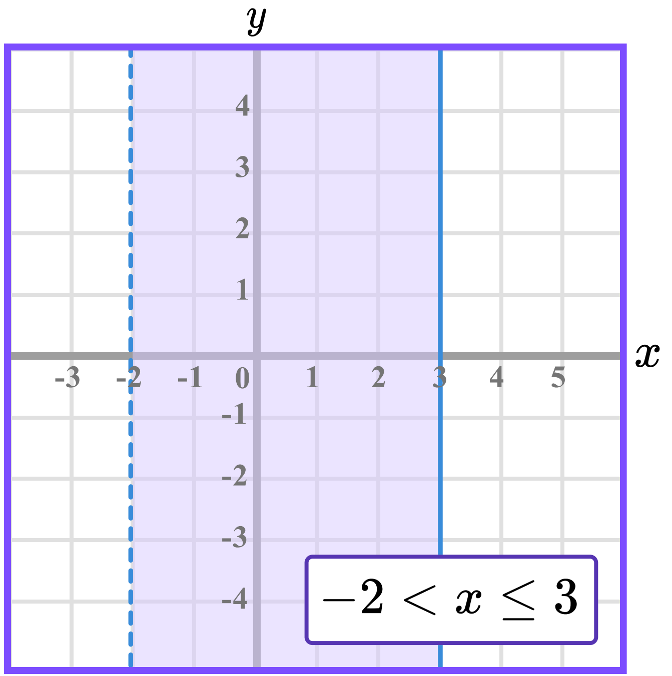
To solve $x^2 + 6x + 5 > 0$, we find the x values when the graph is greater than 0 (above the x axis)
The solution is written as $x > -1$ and $x < -5$

To solve $x^2 + 6x + 5 < 0$, we find the x values when the graph is less than 0 (below the x axis)
The solution is written as $-5 < x < -1$

Inequalities on a Graph

When plotting **inequalities on a graph**, you may be asked to shade regions on the graph to satisfy one or more inequalities.

A **dashed line** means the line is **not included** ($<$/$>$)
A **solid line** means the line **is included** ($\leq$/$\geq$)



Example $-2 < x \leq 3$

We use a **dashed** line for $x = -2$ and shade the region to the **right** of the line because we want the x values greater than -2

We use a **solid** line for $x = 3$ and shade the region to the **left** of the line because we want the x values less than or equal to 3

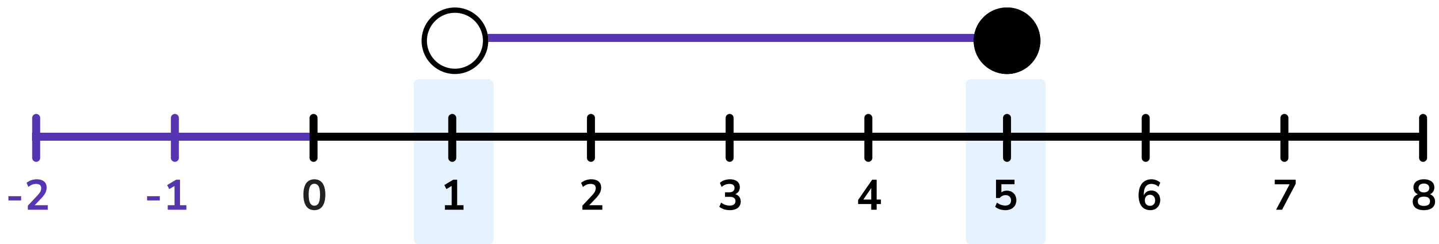
Inequalities on a Number Line

To represent **inequalities on a number line** we show the range of numbers by drawing a straight line and indicating the end points with either an open circle or a closed circle.

An open circle shows that the value is **not included** ($<$ / $>$)

A closed circle shows that the value is **included** ($\leq$ / $\geq$)

Example $1 < x \leq 5$



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