



**THIRD SPACE  
LEARNING**

# Math Intervention Pack

Using multiplication and  
division facts up to  $12 \times 12$

**Grade 3**

## How to use the resources

### 1. Title Slide

Use this slide to activate prior knowledge needed for lesson. Students should be encouraged to initially attempt the question presented independently.

### 2. Let's Learn

Use this slide to introduce the concept. Tutors should work with the student to explore the concept together, usually using diagrams to support understanding.

### 3. Follow Me + Your Turn

The tutor should work through the follow me slide, modeling the process and explaining their thinking out loud.

Students should use the your turn slide as an opportunity to work through a question similar to the follow me questions. They should apply the method modeled by the tutor in the follow me slide. Students should be encouraged to explain their thinking out loud.

### 4. You Do

Students should work through a range of questions that build in complexity.

Tutors can offer support but students should initially be encouraged to attempt these questions independently.

### 5. Go Further

Use this slide to allow students to apply their understanding to a more challenging question in an unfamiliar context.

### 6. Support for Slides

The support slide is used to support students during the lesson. In the tutor notes, there will be guidance as to when to use the support slide.

### 7. Check Your Understanding

Tutors should use this slide to assess the student's knowledge and whether or not they have mastered the concept within the lesson.

## Standard

**3.OA.7:** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

## Key Mathematical Ideas

1. To develop strategies to solve multiplication facts that are not yet memorized
2. To use arrays and area models to solve multiplication facts
3. To use known multiplication facts as tools to solve unknown multiplication facts by partitioning

## Overview

### Terminology

- **Partition:** To express a whole in terms of its parts
- **Associative Property of Multiplication:** An extension of the commutative property; to change the order and group two factors to find the convenient products (such as 10) in order to make the multiplication easier. Students may begin to use parentheses at this level. EX.  $7 \times 8 \times 5 = 7 \times (8 \times 5) = 7 \times 40 = 280$
- **Commutative Property of Multiplication:** Reversing the order of the factors does not change the product. EX.  $8 \times 5 = 40$  and  $5 \times 8 = 40$ , therefore the product of  $8 \times 5 = 5 \times 8$
- **Distributive Property of Multiplication:** Multiplying a sum by a given number is the same as multiplying each addend by the number and then adding the products
- **Product:** The result when two numbers are multiplied
- **Factor:** One of the numbers multiplied to find a product
- **Multiple:** The result of multiplying a whole number by other whole numbers
- **Regular Polygon:** A closed plane figure having three or more sides, where all sides are equal and all angles are equal
- **Perimeter:** Distance around a figure or object

## Overview

### Sentence Stems

- What times ..... is .....?
- How many groups of ..... is .....?
- ..... groups of how many is .....?

### Common Misconceptions

Common Misconceptions	Tutoring Strategies	Checks for Understanding
Students not understand the inverse relationship between multiplication and division	Students can often see lessons like this as just a set of fill in the blank questions without understanding the relationship between multiplication and division facts, and how they can be used to help solve problems we don't know. Remind students of fact families and write out the facts within the fact family of the multiplication or division problem you are discussing with the student to show how if we know one of the facts in the fact family, we can figure out the others. Example of a fact family: $4 \times 3 = 12$ , $3 \times 4 = 12$ , $12 \div 4 = 3$ , $12 \div 3 = 4$	Ex. Because we know $7 \times 5 = 35$ , what would $35 \div 7$ be?
Students not demonstrating fluency with multiplication facts	Discuss strategies that students can use when they get stuck on a problem. They can think of other multiplication facts they do know that are close to the one they want to solve and either build off of those known facts or take away from them. These strategies are shown on the Support slide, so use this if you find that students are struggling to think of ways to solve unknown facts	What other facts do you know that could help you solve this one? What other multiples of ..... do you know that are close to this one? How can we use that to help us?



## Title Slide

### If stuck

- Ask the student, “What comes to mind when you see these three numbers grouped together?” Hopefully this will elicit the response “I think about how  $4 \times 7 = 28$ ”
- If the student is able to recognize the multiplication fact, have a conversation with them about the names of each part of a multiplication problem (some parts have multiple names—no pun intended!)
- 28 is the product of 4 and 7 and it is also a multiple of 4 and 7
- 4 and 7 are both factors of 28
- The above two statements demonstrate to students the math language used to describe multiplicative relationships/proportional relationships to students. Have them practice saying statements like this so they solidify precise mathematics language

### Answers

- 4 is a factor
- 26 is a multiple and product
- 7 is a factor

## Let's Learn

### If stuck

- The goal for this slide is to look for fluency with these multiplication facts and assess students' ability to recall multiplication facts
- The array on this slide is to demonstrate how we know that  $8 \times 9 = 72$  and is not necessarily here as a tool to solve  $8 \times 9$
- " $8 \times 9 = 72$ ; we know this because an 8 by 9 array of counters contains 72 counters"
- The Support Slide shows strategies to solve part a and b on this slide and can be used to support students in automaticity of naming multiplication facts

### Questions

- What is a way that helps you remember  $6 \times 6$ ?  $3 \times 12$ ?
- What times 5 is 45? How many groups of 5 is 45?

### Watch out for

- Students not answering these multiplication facts fluently

### Answers

- a.  $6 \times 6 = 36$
- b.  $36 = 3 \times 12$
- c.  $9 \times 5 = 45$

## Follow Me

### Modeling prompts

- Start by explaining to students that multiplication and division have an inverse relationship, meaning we can answer basic division facts by filling in the missing factor of a multiplication fact we know
- For instance, 35 divided by 7 is the same as thinking of 7 times a missing factor that is equal to 35
- Students may be able to fill in the blank automatically here, but if not, prompt them by asking questions like, “Thinking of your 7 facts, 7 groups of what number equals 35?”
- Part a uses some connections to measurement and geometry. Discuss with students what a regular polygon is a shape with all equal sides. Ask the students how many sides an octagon would have. (It is okay to explicitly tell them what a regular polygon is or how many sides an octagon has as this is not the focus of this lesson—but please draw a picture of the shape to help the student understand what it looks like if they don’t already know what it is!)
- Now that we know this is an 8 sided shape with all equal sides, discuss perimeter with students. Perimeter is the distance around the shape, or the sum of all the side lengths. Since we know the total of all the side lengths, and we know that all the side lengths are the same length, ask students what they think we can do to figure out the length of each side
- You can start by filling out the multiplication problem on the bottom with the parts you know to help students think of the equivalent division problem; e.g.  $8 \times \dots = 48$  therefore,  $48 \div 8 = \dots$

### Answers

- $7 \times 5 = 35$  so  $35 \div 7 = 5$
- $48 \div 8 = 6$
- $6 \times 8 = 48$

## Your Turn

### If stuck

- Use the Support Slide goes over a strategy for remembering  $9 \times 12$  if students are unable to answer this automatically—use this after they find the answer though, and use the suggestion below to help them solve it
- A strategy to use to find the missing factor in  $9 \times \dots = 108$  is to ask students what multiple of 9 they know already that is really close to 108. Most students will answer 90 or 99. Ask them how many groups of 9 their answer is away from 108
- You can start by filling out the multiplication problem on the bottom with the parts you know to help students think of the equivalent division problem; e.g.  $8 \times \dots = 40$  therefore,  $40 \div 8 = \dots$

### Questions

- What is a regular polygon?
- How many sides does an octagon have?
- 7 groups of how many equals 35?
- 9 groups of how many equals 108?
- 8 groups of how many equals 48?
- 8 groups of how many equals 40?
- What multiple of 9 do you already know that is really close to 108? How many more groups of 9 do we need to get to 108?

### Watch out for

- Students struggling to use the inverse of multiplication facts to solve division facts
- Students not demonstrating fluency with multiplication facts

### Answers

- $9 \times 12 = 108$  so  $108 \div 9 = 12$
- $40 \div 8 = 5$
- $5 \times 8 = 40$

## You Do

### If stuck

- Remind students that perimeter means the distance around the shape, or the sum of all the side lengths. When we are working with a regular polygon, all side lengths are the same, therefore we can use multiplication instead of repeated addition to solve. Use the picture of the hexagon to help students recognize how many sides are in a hexagon if they do not already know
- Write out the equation for part b that you are trying to solve; as a multiplicative comparison question, we need to take Ben's time and multiply it by 3 to find Tim's amount.  $8 \times 3 = \dots$
- Write out the missing factor problem to help them solve if they have a difficult time recognizing what the question is asking in part c; "How many groups of 6 eggs would give me 54 eggs?"  $\dots \times 6 = 54$

### Questions

- How many sides does a hexagon have?
- What is a regular polygon?
- What is perimeter?
- How many groups of 6 eggs would give me 54 eggs?

### Watch out for

- Students struggling to use the inverse of multiplication facts to solve division facts
- Students not demonstrating fluency with multiplication facts
- Students having difficulty understanding the context of these word problems and relating them to multiplication/division expressions

### Answers

- a. 42cm
- b. 24 min
- c. 9

## Go Further

### If stuck

- You can start with any blank space where you have 2 pieces of information already. Let students decide where they would like to start; the bottom right corner may be the easiest if they struggle to decide where to start. “How many groups of 7 is 14?” Once they know this, it will most likely be easy to fill in the 9 factor and the 6 multiple from  $3 \times 2 = 6$
- You may want to then shift focus on the 5’s column as that should be fairly automatic for students
- Students will have to figure out the missing factor to  $3 \times \dots = 24$  to get the 8 factor. Build off of 3’s facts they do know if they struggle with this
- Once they have the 8 factor, they can figure out  $8 \times \dots = 48$ . Again, build off of 8’s facts they already know, most likely  $8 \times 5$
- Once they have the 6 factor, they can fill in the missing multiples of 6 in the 5 and 2 columns
- The last thing they will need to do is complete the 8’s column. Use facts they know to build off of or take away from to help them figure them out if they do not know them automatically

### Questions

- What times  $\dots$  is  $\dots$ ?
- Now that I have this missing piece of information, what else can I solve?
- Where is a good place to start?
- What other multiplication fact do I know really well that will help me solve this problem?

### Watch out for

- Students struggling to use the inverse of multiplication facts to solve division facts
- Students not demonstrating fluency with multiplication facts

## Go Further

### Answers

x	5	8	2
3	15	24	6
6	30	48	12
9	45	72	18
7	35	56	14

## Support for Slide(s)

This slide supports the Let's Learn and Your Turn slides.

### If stuck

- Follow the prompts and visuals on this slide
- For part a, make sure students see that the array shows 5 groups of 6 with one additional group of 6

### Questions

- What is one more 6 than 5 groups of 6?
- What is one more 12 than 2 groups of 12?
- What is one less 12 than 10 groups of 12?

### Answers

- $6 \times 5 = 30$  so  $6 \times 6 = 30 + 6 = 36$
- 36
- 108

## Check Your Understanding

### Correct answers

- 9
- $7 \times \dots = 63$
- $7 \times 9 = 63$



Today you will learn about



# Using multiplication and division facts up to $12 \times 12$

Warm-up question

What is the relationship between these three numbers?  
(Hint: Terms can be used more than once, and one number might have more than one term!)

4

28

7

Factor

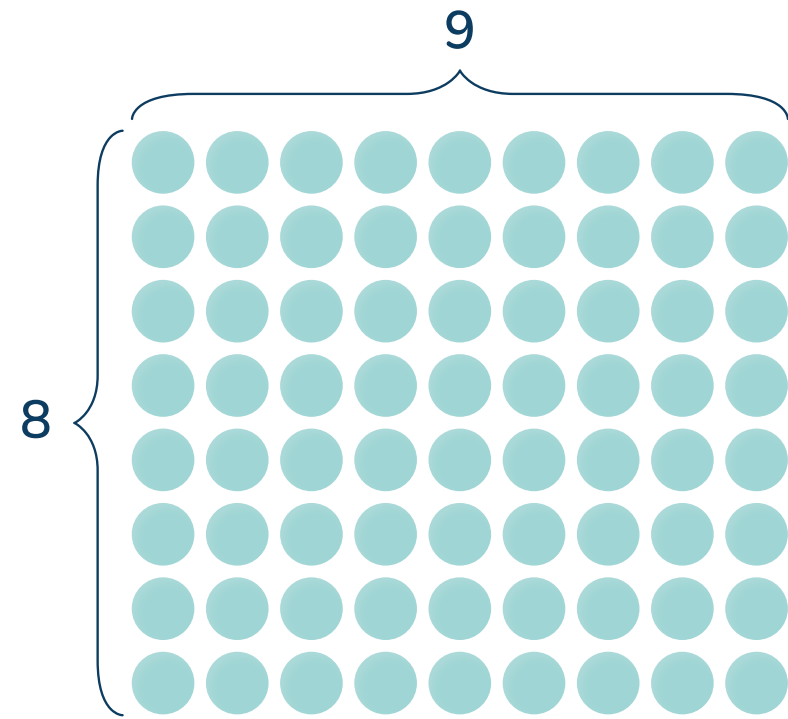
Multiple

Groups of

Product

# Let's learn

Knowing your multiplication facts can help you solve simple problems.



$$8 \times 9 = 72$$

↑  
This is the product.

72 is a **multiple** of 8 and 9

8 and 9 are **factors** of 72

Remember, we can multiply in either order.

$$8 \times 9 = 9 \times 8$$

The product will remain the same.

This is called the **Commutative Property of Multiplication**

Can you recall the following multiplication facts?

a  $6 \times 6 =$  .....

b .....  $= 3 \times 12$

c .....  $\times 5 = 45$

d Which of these numbers are multiples of 3?

16

18

23

9

24

## Follow me



We can use multiplication facts to solve division problems.

$$35 \div 7$$

$$7 \times \text{.....} = 35 \text{ so } 35 \div 7 = \text{.....}$$

a Let's solve a problem using our multiplication and division facts.

A regular octagon has a perimeter of 48 cm.  
What is the length of each of the sides?

$$\begin{array}{r} \text{.....} \bigcirc \text{.....} = \text{.....} \\ \text{.....} \times \text{.....} = 48 \text{ cm} \end{array}$$

## Your turn



Use your multiplication facts to solve the following problem.

$$108 \div 9$$

$$9 \times \text{.....} = 108 \text{ so } 108 \div 9 = \text{.....}$$

a Solve the following problem.

8 children spend a day washing cars  
and earn \$40 altogether.  
If they share the money equally  
how much do they each get?

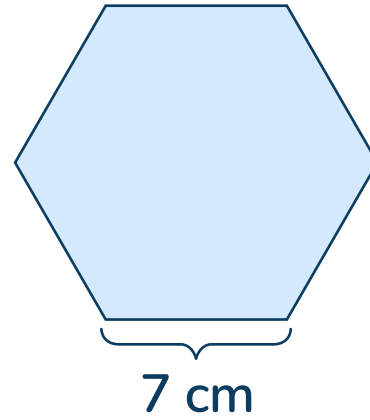
$$\begin{array}{r} \text{.....} \bigcirc \text{.....} = \text{.....} \\ \text{.....} \times \text{.....} = \$40 \end{array}$$

a

A regular hexagon has sides of 7cm.

What is its perimeter?

.....



b

It takes Ben 8 minutes to walk to school.

It takes Tim 3 times as long.

How long does it take Tim to walk to school? .

.....

c

An egg carton contains 6 eggs.

I need 54 eggs.

How many cartons should I buy?

.....

Go further

Can you complete this multiplication grid using your multiplication and division facts?

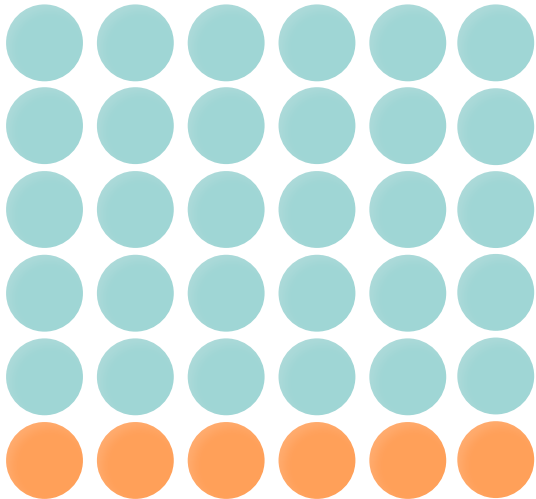
x	5	.....	.....
3	15	24	.....
.....	.....	48	.....
.....	45	.....	18
7	.....	.....	14

We can use the facts you do know to help you figure out the multiplication facts you are stuck on.

a

$6 \times 6$

Drawing an array can help you see other facts you know within it.



Do you know 6 times 5?

$6 \times 5 =$

.....

So  $6 \times 6$ :

$+ 6 =$

.....

.....

b

Do you know  $2 \times 12$  ?

So  $3 \times 12$  will be 12 more.

.....

c

Do you know  $10 \times 12$  ?

So  $9 \times 12$  will be 12 less.

.....

## Check your understanding

Solve the division fact using a multiplication fact you know:  $63 \div 7$

Why do I need to try this question on my own first?

- To show your tutor what you understand
- To give you more practice
- To show your teacher how you are doing



## Do you have a group of students who need a boost in math?

Each student could receive personalized lessons every week from our specialist one-on-one math tutors.




- ✓ Differentiated instruction for each student
- ✓ Aligned to your state's standards
- ✓ Scaffolded learning to close gaps

“We just had our first session and it went great! The kids really liked it and felt like they were learning! One even said he finally felt like math was making sense.”



Michelle Craig, Instructional Coach,  
Sherwood Forest Elementary, Washington

## Speak to us

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