



**THIRD SPACE
LEARNING**

Guide To Hands On Manipulatives

15 hands on resources every elementary classroom
should have - and how to get them on a budget

School and District Administrator Guides

Contents

Introduction	2
Storage and management of math manipulatives	2-3
Base Ten blocks	4-5
Place value counters	6-7
2-sided counters	8-9
Counting cubes	10-11
Hundreds chart	11-12
Ten frames	13-14
Pattern blocks	15-16
Fraction tiles	16-17
Fraction circles	18-19
Cuisenaire rods	20-21
Geoboards	22-23
Playing cards	24-25
Dice	26-27
Drinking straws	27-28
Food	29-30
Extra reading and support	31
References	31

Introduction

Math manipulatives refer to the hands-on resources used in math classrooms to develop children's understanding of mathematical concepts, often in a practical, tactile way.

Although most of us now understand the importance of math manipulatives in the classroom, attitudes towards them have changed considerably over the past ten years. It was not long ago that these concrete resources were relegated to Pre-K and kindergarten classrooms only.

With an increasing number of schools adopting a mastery approach to teaching math, manipulatives are now a common feature of math lessons across all classrooms in elementary schools - and even middle and high school.

However, school teachers and leaders often have questions about the usage and management of the resource, particularly which manipulatives will have the most impact, which are worth the investment, and, perhaps most importantly, how you can make your classroom resources budget stretch even further.

So, to help inform your decisions, here's our rundown of the top 15 math manipulative resources that are essential to any math classroom.

With ever tightening school budgets, we also look at alternative options, including online and do-it-yourself options, and provide you with example activity ideas to give you an idea of the versatility these resources can offer.

Storage and management of manipulatives

The key in managing manipulatives is for them to be easily accessible, while ensuring they don't cause a distraction for some children. Decide how you are going to store your resources and give clear expectations for your class to follow, including when and how these resources can be used.

You may choose to store all resources together in a location accessible to all students. Alternatively, you may decide to put together table boxes, each containing the resources, which can be placed in the center of each table during math lessons.

Initially, children will need a lot of support and guidance on which manipulatives to select and how to use them. Over time, children can be guided towards making their own choice of manipulatives - this allows them to take ownership of their own learning and development.

Importantly, math manipulatives do not have to be concrete resources. Consider bringing manipulatives into the classroom through lesson slides and your interactive whiteboard to both introduce new manipulatives and to reinforce manipulatives your students may be more used to physically handling. In this guide, you can also find online alternatives to hands-on resources.



In Third Space Learning online lessons, we use a wide variety of visuals to mirror resources used in the classroom. By following a Concrete - Representational - Abstract approach and using images of resources that are familiar to children, we are able to develop and deepen their understanding of math through manipulatives, even through a screen.

Throughout this resource, the examples you'll see of math manipulatives in action are taken from the lessons used in our online one-to-one math tutoring sessions.

Let's learn

100 can be thought of as a bundle of **ten tens** — called a “**hundred**.”

Let's count in steps of 10 to 100.

a

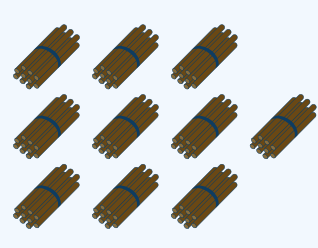
10	10	10	10	10
10	10	10	10	10

How many steps of ten did you count?

.....

b

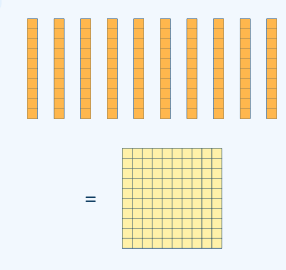
There are 10 sticks in each bundle.



How many sticks are there altogether?

.....

c



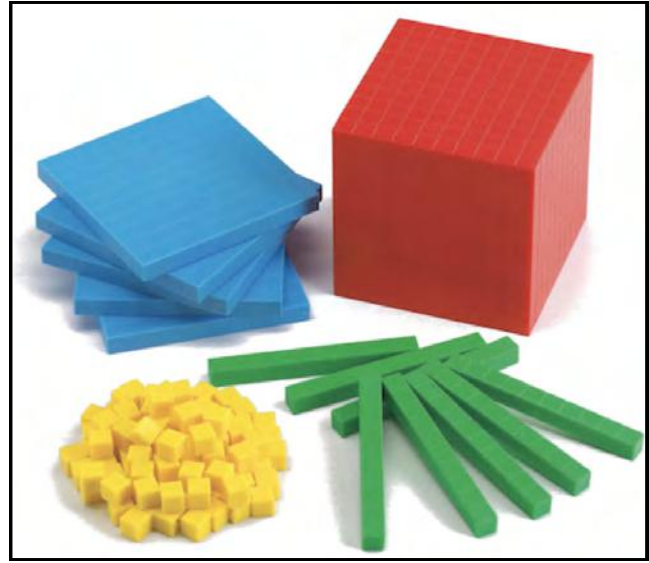
=

..... tens are **equal** to 100

Grade 2 - Knowing that 10 tens are equal to 100

An example from a Third Space Learning lesson where a range of manipulatives are used to introduce and consolidate the concept of hundreds.

Base ten blocks



Summary

- ✓ Base ten blocks are an essential manipulative in any school.
- ✓ They are usually plastic (although some schools may still have the older, wooden versions), consisting of ones cubes, tens rods, flat hundred squares and thousand blocks, which children can use to build and represent numbers.
- ✓ They are a fantastic visual way for children to understand the relationship between ones, tens, hundreds, and thousands.

Grade level

Base ten blocks are predominantly used in grades K-2, but some older children who are struggling with the concept of place value may benefit from using them. They are also useful in decimal representation in 5th grade and we use them for this purpose in our online tutoring sessions.

As numbers get larger and children become more confident with the base 10 number system, this resource is usually replaced with place value counters.

Topics

This resource is brilliant for children being introduced to place value and the base 10 number system. Children begin by using the base ten blocks to represent numbers. They are also essential for children when they are first introduced to formal written methods of addition, subtraction, multiplication, and division.

Base ten blocks are particularly useful for helping children understand the concept of grouping / exchanging, as children are able to physically exchange the blocks, for example, ten ones cubes for a ten rod, or a ten rod for ten ones cubes.

Cost

Small sets can be purchased for approximately \$25 and whole class sets for roughly \$90-\$100.

Online alternative

Didax: Virtual Base 10 Blocks, Oryx Learning: Virtual Base 10 Blocks¹

Do it yourself

Straws are an excellent alternative to base ten blocks (they are also considerably cheaper!). One straw represents 1, a bundle of 10 straws represents 10, and a bundle of 100 straws represents 100.

Keep in mind that straws are better suited when working with tens and ones, as larger numbers require a huge number of straws.

Example activity: Addition of 4 digit numbers - standard algorithm

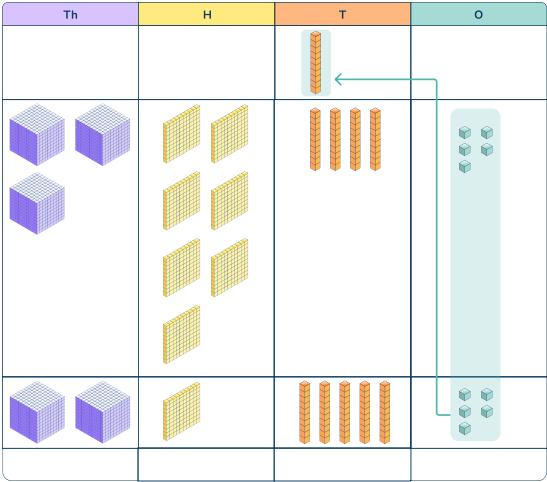
Set the base ten blocks out on a place value chart to represent the thousands, hundreds, tens, and ones of two numbers and set them out in the standard algorithm method. The ones can be added, then the tens, followed by the hundreds and finally the thousands. If any columns add up to greater than 9 (9 ones, 9 tens and so on), children are able to physically regroup tens, for example ten ones cubes for a tens rod, and place this beneath the tens column.

Let's explore this more

Let's look at showing this problem using Base 10 blocks.

a

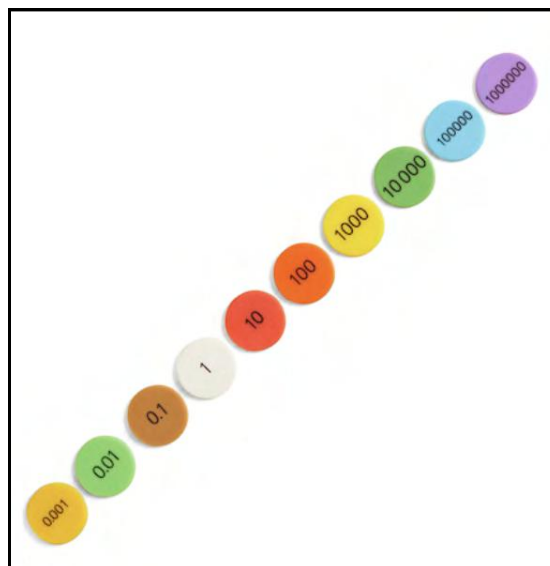
	Th	H	T	O
	3	7	4	5
+	2	1	5	5



Grade 4 - Adding 4-digit numbers using the standard algorithm

Base ten blocks are used to help students understand what is happening when numbers are added using the standard algorithm. It shows how we regroup 10 ones for 1 ten.

Place value counters



Summary

- ✓ Place value counters are the next step in the progression from concrete Base 10 towards a more abstract understanding of place value.
- ✓ They serve a similar purpose to the base ten blocks, but with the key difference being that they are all the same size.
- ✓ This enables students to work with much larger numbers, in addition to progressing on to decimal numbers.
- ✓ They also ensure children are not over-reliant on being able to see the difference in size to understand the concept of the base 10 number system.

Grade Level

Place value counters are an essential resource for children in grades 2-5, although some schools may choose to introduce them in 1st grade.

Topics

Place value counters are very useful for children to understand the concept of place value and can be used for much larger numbers than the base ten blocks.

In addition to the place value of whole numbers, they are also useful for children's introduction to decimal numbers. Place value counters are also very helpful for children learning the standard algorithms of addition, subtraction, multiplication, and division.

Cost

Class sets cost approximately \$100. Smaller sets can be purchased for roughly \$25.

Online alternative

MathsBot Place value counters² (includes large whole numbers and decimal numbers)

Do it yourself

Bottle tops are an excellent free, sustainable way to make your own place value counters. You could ask parents to collect bottle tops in a range of colours. Using a permanent marker, you can then make your own place value counters.

If you are short on bottle tops, a cheap alternative is to buy a set of plain, colored counters. These are much cheaper than buying the commercial place value counters and can also be written on, using a permanent marker. Alternatively, you could make your discs out of cardstock.

Example activity: Division with 1-digit divisors

Place value counters can be set out to make the dividend in a division equation. For example, 428 is made with 4 hundreds counters, 2 tens counters and 8 ones counters. If this number is being divided by 4, for example, the counters in each column are then grouped into groups of 4 for children to work out how many groups of hundreds, how many groups of tens and how many groups of ones they have. This then gives them their final answer, 107.


Let's learn


We use the **partial quotient method** when dividing by a one-digit number or dividing by a number you know your multiplication facts for, for example, 12's.

$84 \div 4$

4

Although we can probably do this mentally, let's look at the partial quotient method.

The partial quotient method always starts with looking at your **dividend** partitioned into place value parts.

$8 \text{ tens} \div 4 =$
 $4 \text{ ones} \div 4 =$

tens

 ← Write the answer in the tens column

one

 ← Write the answer in the ones column

We can write the answers in the place value columns because there was no regrouping involved in this particular problem.

Grade 4 - Dividing using the partial quotient method

In this example, the place value counters help students to understand division and why we say '4 goes into 8' twice. What we mean is that we can make two groups of 4 (hundreds) and we have 0 hundred left.

2-sided counters



Summary

- ✓ 2-sided counters are a simple but very effective resource.
- ✓ They are plastic counters with one color on one side and a different color on the other.
- ✓ They have a diverse range of uses and can be used to support a wide range of math concepts across all grade levels.

Grade Level

2-sided counters are one of the few resources which are useful all the way from Pre-K through grade 5.

Topics

2-sided counters have a wide range of uses, including place value, computations, fractions, ratio and algebra.

Cost

\$10 for a pack of 200 counters.

Online alternative

[Didax 2-color counters](#)³

Do it yourself

You can make your own 2-sided counters by cutting discs out of cardstock and painting either side a different color. Another option is to ask children to bring in a range of small pebbles and paint either side of the pebble in a different color.

Example activity: Fractions of amounts

2-sided counters are great for investigating fractions of amounts. Early activities can include identifying what fraction of a set of 2-sided counters are red and what fraction are yellow. They can also be used alongside the bar model to investigate more challenging fractions of amounts. For example, $\frac{3}{5}$ of 25. Children can draw a bar model split into fifths.

They can then share the 25 counters across the 5 sections. Counting the counters in each fifth section will give them $\frac{1}{5}$ of 25. Adding the counters in three of the fifth sections, or alternatively multiplying their answer to $\frac{1}{5}$ by 3, will give them their final answer of 15.


Let's learn


The **distributive property** states that multiplying a sum by a given number is the same as multiplying each addend by the number and then adding the products.

Let's look at the following problem.

7×6

a First, partition one of the factors using easier numbers to work with.

7

$\swarrow$
 $\searrow$

5

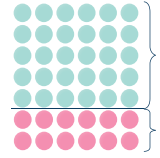
+

2

Multiplying numbers by 5 and 2 is easy to do in our heads.

Therefore, we can think of 7×6 as $(5 + 2) \times 6$

b Using the array, we can see how 7 groups of 6 can also be seen as 5 groups of 6 and 2 groups of 6.



$\left. \begin{array}{c} \text{5 rows} \\ \text{2 rows} \end{array} \right\} \begin{array}{l} 5 \times 6 \\ 2 \times 6 \end{array}$

 $\left. \begin{array}{c} \text{5 rows} \\ \text{2 rows} \end{array} \right\} 7 \times 6$

Therefore, $7 \times 6 = (5 \times 6) + (2 \times 6)$

$= 30 + 12$
 $= 42$

Grade 3 - Using the distributive property to multiply

Here, double-sided counters are used to show an array for 7×6 . This shows that we can see 7×6 as $(5 \times 6) + (2 \times 6)$

Counting cubes



Summary

- ✓ Counting cubes consist of different colored, plastic cubes.
- ✓ Each cube connects to another to build a stack, allowing students a multi-sensory way to develop number sense.
- ✓ Some sets of counting cubes have the ability to make connections on all four sides of each cube, allowing practice for more complex math content.
- ✓ These may also be referred to as snap cubes or counting blocks, among other names.

Grade Level

Counting cubes are typically used in classrooms in grades K-2 as students build number sense and place value understanding. However, counting cubes can also be used in grades 3-5. For example, these are a great resource for teaching volume in grade 5.

Counting cubes can be used in all grade levels as an intervention resource.

Topics

Counting cubes can be used to teach a wide range of concepts, such as: counting, place value, addition and subtraction, grouping, fractions, patterns, and volume.

Cost

There are many different types of sets of counting cubes available, ranging in quality. Therefore, a set of 1,000 counting cubes can range from \$25 to \$200.

Online alternative

Didax Virtual Manipulatives: Unifix cubes⁴

Example Activity: Volume

Counting cubes are great for helping 5th graders understand volume. Fifth graders will especially enjoy an activity where they get to “play” with cubes, as manipulatives are not used as often in upper elementary grades as they are in the lower grades. For an easy volume activity, give students a bunch of cubes to use individually, with a partner, or in a small group. Give students a number (such as 12) and ask them to make a figure out of their cubes with a volume of 12. Students will see that the cubes can be constructed in different ways and still have the same volume.

Hundreds chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

 THIRD SPACE LEARNING

Summary

- ✓ A hundreds chart, also called a 100 chart or hundred chart, is a chart with the numbers 1-100 written in columns and rows. Some newer charts go up to 120 instead of 100.
- ✓ Often, a hundreds chart will be color-coded to show multiples, or odd numbers will be one color and even numbers another color.
- ✓ Hundreds charts are also used to help students practice multiplication facts by skip counting.

Grade Level

Hundreds charts can be used in all grade levels and will often be seen posted in a classroom. They may be used more often in lower grades and in 3rd grade as students begin multiplication.

Topics

Hundreds charts are a good visual resource to help students with counting, adding, multiplication, patterns, skip counting, prime and composite numbers, and more.

Cost

Individual poster size hundreds charts cost approximately \$4.

Online alternative

Didax 120 board⁵

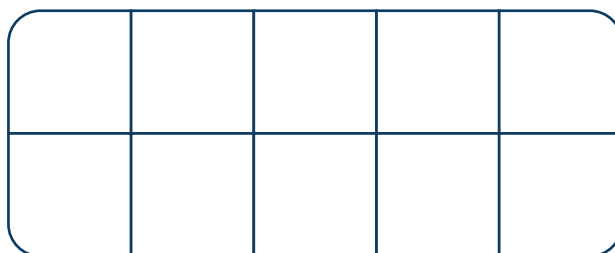
Do it yourself

Hundreds charts are very easy to create yourself for little to no cost. There are many free, printable charts available online. Alternatively, you could give students paper and a ruler and ask them to create their own.

Example activity: Look for patterns in multiples.

For this activity, students should get a blank hundreds chart, a plastic sheet protector, and a dry erase marker. (Note: If you have no sheet protectors available, print multiple smaller hundreds charts on one or two pages for students, or allow students to use an online/virtual hundreds chart.) Give students a number, such as 5, and ask them to find all of the multiples of 5 on the chart and color them in with their dry erase marker. Then, ask students to notice what patterns they notice. (All numbers end in 0 or 5, etc.)

Tens frames



Summary

- ✓ Tens frames are rectangular frames split into ten sections.
- ✓ Thinking about numbers using a tens frame can be a helpful way for children to learn basic number facts.
- ✓ They help children to visualize numbers within ten and beyond and are a great tool for helping to develop number sense.
- ✓ For older children, tens frames are useful for helping to understand and visualize decimal numbers, including addition and subtraction of decimals.

Grade Level

Tens frames are useful across all grade levels from Pre-K to grade 5.

Topics

For younger children, tens frames are a great resource for counting, place value, addition / subtraction of numbers within 10 and 20.

For older children, the value of the tens frame becomes one, with each section representing 0.1. Using the manipulative in this way enables children to visualize and understand decimal numbers, in addition to helping to visualize addition and subtraction of decimal numbers.

Cost

Class sets cost \$30-\$40 and smaller sets can cost around \$10.

Online alternative

[Didax Tens Frames](#)⁶

Do it yourself

Tens frames are one of the easiest manipulatives to make yourself. Either draw the tens frame grids, print tens frame templates from the internet, or use empty egg boxes. These can be used with counters, or as a cheaper alternative, colored pasta, cereal pieces, coins or anything you can get your hands on!

Example activity: Number bonds

Tens frames can be used to help children to learn their number bonds to 10 and to 20 (if two tens frames are used). Children put counters in the frame to make the number and then use different colored counters to make that number up to 10 or to 20. Repeated use of the tens frame helps children to visualize and internalize their number bonds.


Let's learn


100 can be thought of as a bundle of **ten tens** — called a "**hundred**."

Let's count in steps of 10 to 100.

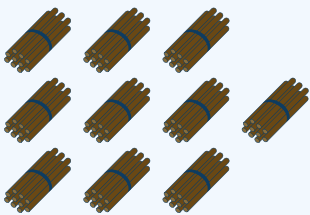
a

10	10	10	10	10
10	10	10	10	10

How many steps of ten did you count?

.....

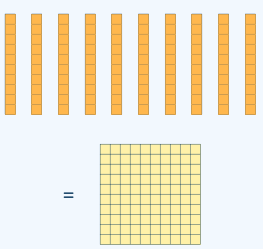
b There are 10 sticks in each bundle.



How many sticks are there altogether?

.....

c



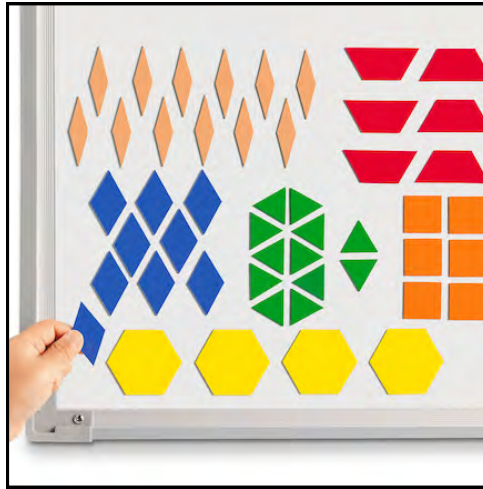
tens are **equal** to 100

.....

Grade 2 - Knowing that 10 tens are equal to 100

In this example, we use tens frames where the value of the tens frame is 100. This helps link knowledge of 10 ones equally 10 to 10 tens equally 100.

Pattern blocks



Summary

- ✓ Pattern blocks are a set of shapes (usually plastic or foam) that include hexagons, squares, triangles, trapezoids, rhombuses, and parallelograms. Each shape has its own color.
- ✓ The shapes are designed to fit together as they all have sides of the same lengths - the one exception to this is the trapezoid, which has one long side, but this length is twice the length of the smaller side.
- ✓ Pattern blocks are a common classroom manipulative for all grade levels and are typically made of wood, plastic, or foam.

Grade Level

All elementary grade levels can benefit from pattern blocks in the classroom.

Topics

Pattern blocks can be used in grades K-2 for shape recognition, grouping and sorting, and patterns. In grades 3-5, pattern blocks can be used when learning fractions, 2D shapes, area and perimeter, symmetry, angles, and more!

Cost

Pattern blocks can be purchased in a variety of quantities and materials. A classroom set of pattern blocks can be purchased for \$120 and smaller sets can be purchased anywhere from \$10-\$30.

Online alternative

[Didax virtual pattern blocks⁷](#)

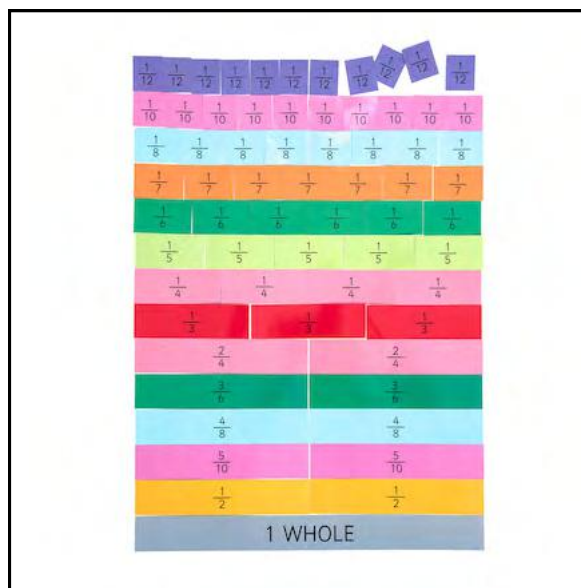
Do it yourself

There are printable pattern blocks available online for free or low cost. Alternatively, students could make their own set out of colored paper, using rulers to ensure sides remain the same length.

Example activity: Adding unit fractions

An excellent way to use pattern blocks is when introducing fractions in 3rd grade. Give each student, pair of students, or small group, a small set of pattern blocks. (Note: they will only need enough to make one whole out of each shape. So, 1 hexagon, 2 trapezoids, 3 rhombuses, etc.) Show students the yellow hexagon and tell them it represents one whole. Then, give them the task of creating one whole out of each of the other shapes, then writing an addition equation to show how the pieces equal one whole. (2 trapezoids would be $\frac{1}{2} + \frac{1}{2} = 1$) As a challenge, you can ask students to create one whole using two or more shapes.

Fraction tiles



Summary

- ✓ Fraction tiles, or fraction bars, are a very visual, hands-on resource which enables children to explore fractions.
- ✓ Without the support of concrete resources, fractions are a very abstract concept which some children can find difficult to grasp.
- ✓ This resource consists of a series of 'tiles' arranged into rows. Each row is split into a different fractional amount and adds up to one whole.

Grade Level

This resource is suited for grades 3-5.

Topics

Fraction tiles have a wide range of uses. From initial understanding of what a fraction is, through to finding equivalent fractions, operations with fractions, and mixed number / improper fractions.

For 3rd grade, only part of the resource can be used, since this grade level is limited to fractions with denominators 2, 3, 4, 6, and 8. As children move further up in school, they will use larger and more difficult denominators. This resource is great for helping children to not only understand what fractions are but also how they are all linked.

Cost

One set of towers is approximately \$10-\$15. Two children can share a set, but larger groups wouldn't be able to use the resource effectively.

Online alternative

[Didax fraction tiles](#)⁸

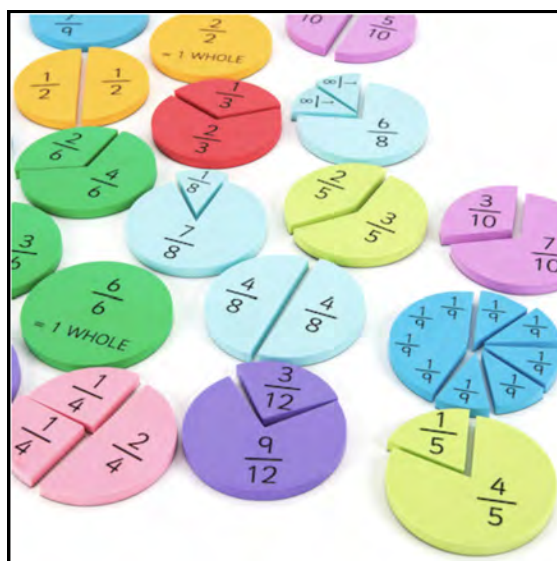
Do it yourself

You can make a paper or cardstock version of the tiles, by either printing out and laminating fractions strips off the internet or by making your own from scratch.

Example activity: Equivalent fractions

Children use the fraction tiles to compare different fractions and identify which fractions are equivalent. For example, if identifying fractions equivalent to $\frac{1}{2}$, children can look at which tiles line up exactly below the $\frac{1}{2}$ tile and find all the fractions equivalent to $\frac{1}{2}$ from their set of fraction tiles.

Fraction circles



Summary

- ✓ Fraction circles have a similar function to the fraction tiles.
- ✓ They are a circular resource, usually made from foam or plastic. Each set has a series of circles, split into different fraction amounts.
- ✓ They are also very good for enabling children to explore fractions in a very visual, hands-on way.
- ✓ This resource doesn't all connect as one like the tiles do and the smaller fraction pieces may be difficult to keep track of, but nevertheless, they are still an excellent resource.

Grade Level

Fraction circles are most suited for grades 3-5, although younger children could use some of the circles to explore simple fractions.

Topics

They can be used for a whole range of fraction concepts. For example, understanding basic fractions; finding equivalent fractions; fraction calculations; mixed and improper fractions.

Cost

Individual sets of fraction circles are approximately \$8. These can be used by a pair of students. A larger set of fraction circles can be found for approximately \$40.

Online alternative

Online fraction circles⁹

Do it yourself

You can make your own fraction circles out of paper plates, or print out and laminate sets.

Example activity: Adding fractions

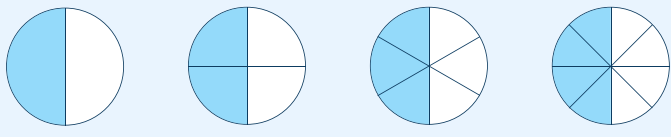
Children use pieces from the fraction circles to make the two fractions they are adding. For example, $\frac{2}{8} + \frac{5}{8}$. They can quickly see that they now have $\frac{7}{8}$. It also helps children to understand the concept of the denominator staying the same when adding and subtracting fractions.


Let's learn


Equivalent fractions are fractions that have different numerators and denominators but have the same value.

Each circle has been split into a different number of parts, but the shaded portion of the whole circle remains the same.

Equivalent fractions
are equal in value.



$\frac{1}{2}$

=

=

=

a Each remaining fraction has a different numerator and denominator than $\frac{1}{2}$, but still represents $\frac{1}{2}$ of the whole circle. Fill in the remaining fractions.

Grade 3 - Understanding equivalent fractions

Fraction circles are used in the lesson above to show the equivalence between $\frac{1}{2}$ and other fractions.

Cuisenaire rods



Summary

- ✓ Cuisenaire rods are plastic or wooden and are color-coded depending on their size (from 1cm to 10cm).
- ✓ They provide an interactive, hands-on way to explore math concepts, such as number bonds, fractions, decimals, and ratio.

Grade Level

A useful resource for all elementary classrooms.

Topics

Cuisenaire rods are very versatile and can be used to teach a wide range of concepts, including: number bonds, patterns, fractions, decimals, and as a concrete aid when using bar models.

Cost

A full classroom set can cost upwards of \$100, but these can be used with smaller groups and partners. Small group sets can be bought for about \$20.

Online alternative

[Mathsbot online Cuisenaire rods](#)¹⁰

Do it yourself

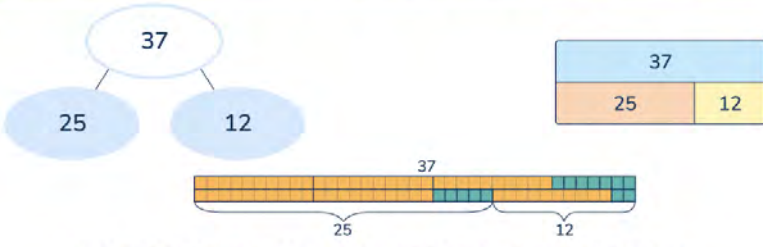
You could cut out and color your own set of flat Cuisenaire rods, using cardstock or laminated paper.

Example activity: Division with remainders

Children can explore division using Cuisenaire rods. For example, for $10 \div 3$, children will take a 10 rod and see how many 3 rods they can put alongside it. They will be able to visually see that three 3s go into 10 and then they will need a 1 rod to make it up to 10. They will then have an answer of 3 remainder one.

Let's learn

By understanding the **relationship** between addition and subtraction, it can help you check your answers and help you solve missing number problems.



Using the part-whole models above, we can write a set of calculations.

a + = 37

c - = 12

b 37 = +

d 25 = -

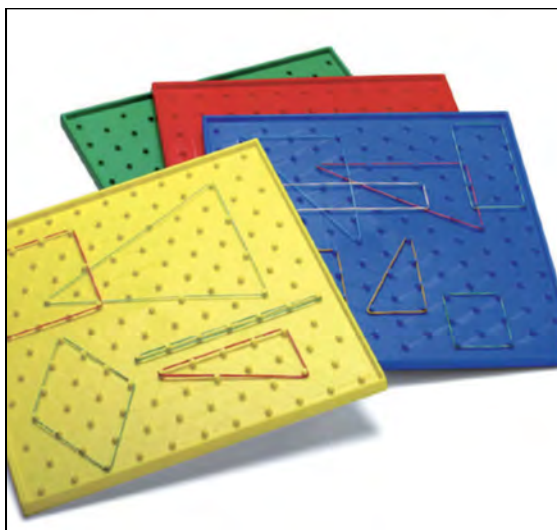
What's the same and what's different?

The = sign does not always have to go at the end of a calculation.

Grade 2 - Using the inverse.

In this example, bar models are used as a representational version of Cuisenaire rods. Tens and ones are used to show the expression $25 + 12$ in a linear way and as a practical model for the bar model shown. It helps students to see that 37 is the total (whole) and then we can rearrange the numbers to come up with a set of equations.

Geoboards



Summary

- ✓ Geoboards are a great way to explore spatial relationships and geometric concepts.
- ✓ They are usually wooden or plastic, with rubber bands stretched over the pegs to make different shapes.
- ✓ They are great for exploring sides and vertices, investigating area and perimeter, symmetry, angles and much, much more.

Grade Level

This resource is more suited to grades 3-5, as manipulating the rubber bands can be difficult for younger students.

Topics

Geoboards can be used to learn about 2D shapes, looking at sides and vertices. They can also be used to learn about angles, area, and perimeter.

Cost

A set of 6 geoboards can be bought for approximately \$17.

Online alternative

[Didax geoboard](#)¹¹

Playing cards



Summary

- ✓ Playing cards are a cheap, but often under-utilized, manipulative in elementary classrooms. They are a good resource to use as they don't just show the number but also provide a visual representation of the number through the symbols on each card.
- ✓ They are great for motivating children, as they link playing cards to games and having fun. Children often forget they are using math when playing mathematical games with cards.
- ✓ Playing cards are a useful addition to the classroom, as not only can they be used in math lessons, but they are a great time filler at other times during the day, if children are taught a range of math games they can play with the cards.

Grade Level

Playing cards are a useful resource in all grade levels.

Topics

In addition to being used for a variety of math games, playing cards can be used to help children understand a range of math concepts, including: place value, addition / subtraction, and fractions.

Cost

A traditional pack of playing cards costs approximately \$3.

Online alternative

[Online playing cards](#)¹²

Do it yourself

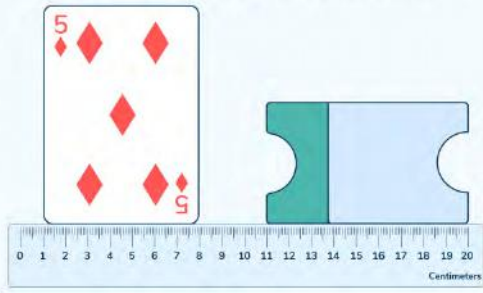
Playing cards are a cheap resource to buy, but if you don't have playing cards available, many of the games can be played using homemade number cards as an alternative. Alternatively, you could print out and cut up your own.

Example activity: Total of 10 card game – number bonds to 10

Using a deck of cards, with the face cards removed, lay out the cards (face-up) in four rows of 5. The aim of the game is for each player to remove two or more cards which add up to 10. The winner is the player with the most cards at the end of the game. This can extend to make 20, with children finding two or more cards which add together to make 20.

Check your understanding

How much longer is the ticket than the card?



The is centimeters longer than the

Why do I need to try this question on my own first?

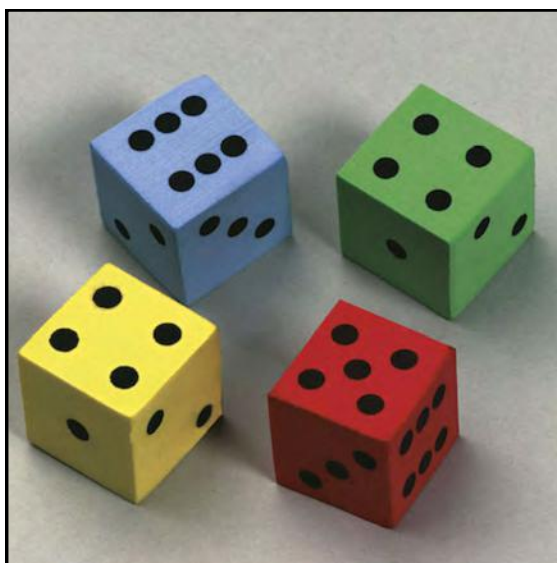
- To show your tutor what you understand
- To give you more practice
- To show your teacher how you are doing



Grade 2 - Comparing lengths of objects

In this lesson we use a playing card and ticket, to help students apply their understanding of measurement to a real life situation.

Dice



Summary

- ✓ Dice are another cheap but effective resource.
- ✓ They are great for engaging children through math games and activities and are particularly good for helping children improve their fluency.
- ✓ They can be purchased with different numbers of sides, giving them more uses than the standard 6-sided dice alone.

Grade Level

Dice are a useful resource for all grade levels.

Topics

Dice can be used to support children with a range of math concepts, such as: place value, fluency with the 4 operations, fractions, and decimals. They are particularly good for simple fluency games and can be used as a time filler at other times during the day.

Cost

A set of 100 mixed dice costs about \$10.

Online alternative

[Online dice resource](#)¹³

Do it yourself

You could make your own dice using modeling clay. Alternatively, you could make a set of number cards to use as an alternative to dice.



Example activity: 'Pig' – for improving fluency in addition

Two players take turns throwing two dice and recording the total on the board. The aim of the game is to be the first player to reach 100 (or 50 for younger players).

Once a player has thrown their dice once, they can choose to throw again, to try to add to their total, or pass the dice on to the other player. The risk is that if they throw a 1 on either dice, they lose all the points scored in that round. If they throw two 1s, they lose all the points scored in the game so far. Players have to decide whether to play it safe or take a risk.

Drinking straws



Summary

- ✓ Drinking straws bundled together are an excellent, cheap alternative to the commercial base 10 resources.
- ✓ They are also a resource that parents can easily replicate at home.
- ✓ They are great for helping children to understand place value and exchanging/regrouping.

Grade Level

Straws can be useful resources for all grade levels.

Topics

Straws can be used individually to represent 1, or bundled into groups to represent 10 and 100. They are good for place value and learning the 4 operations- particularly when regrouping / exchanging is involved, as children are able to physically group the straws together or take them apart when exchanging.

Cost

A pack of 100 straws cost approximately \$5. The best straws to use are the small straws which come with children's school milk!

Example activity: Subtracting two 2-digit numbers using the standard algorithm

Children make the first number in the equation using bundles of ten straws for the tens and individual straws for the ones. They then start with the ones and subtract the ones from the first number. If the second number has more ones, they can't subtract that many ones, so will have to exchange a ten for ten ones. They can physically do this by picking up the bundle of ten straws, removing the elastic band and placing the ten ones in the ones column.


Let's learn


A 2-digit number can be divided by a single-digit number, by **partitioning** the 2-digit number and then dividing each part.

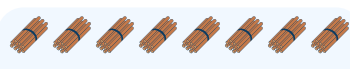
84 sticks are shared equally between 4 children.
How many sticks does each child get?

The calculation is $84 \div 4$

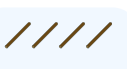
	10s	1s	
4	8	4	

Each child gets sticks.

Partition 84 into 8 tens and 4 ones.



8 tens = 80

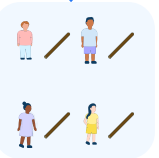


4 ones = 4

$\div 4$



$\div 4$



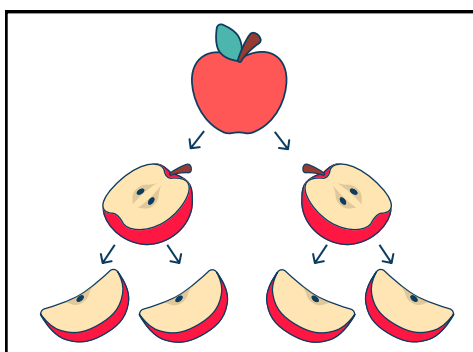
a $8 \text{ tens} \div 4 = \dots\dots\dots \text{tens}$

b $4 \text{ ones} \div 4 = \dots\dots\dots \text{one}$

Grade 4 - Dividing using partitioning.

In this example, rather than straws, we use bundles of sticks to model division where first, we divide the tens and then, we divide the ones. This is a practical way that students can be introduced to division where we start with the tens first.

Food!



Summary

- ✓ Various snack foods can be used as math manipulatives across all grade levels and in many content areas, and the best part is that it is super motivating and fun for kids!
- ✓ Popular snack foods used as math manipulatives are Cheez-its, Skittles, M&Ms, graham crackers, chocolate bars, cereals, and many more!

Grade Level

There are many, many creative ways for teachers to incorporate food into their math lessons as manipulatives in all elementary grade levels.

Topics

Snack foods can be used for a variety of topics. Here are a few example ideas:

Colorful foods such as M&Ms, Skittles, gummy bears, Froot Loops, multi-color goldfish, etc.

- fractions of a set, graphing, grouping, counting, addition and subtraction

Square / rectangular foods such as Cheez-its, graham crackers, saltine crackers, etc.

- area, perimeter, fractions, graphing

Cost

Buying snacks for a classroom isn't always cheap, although it can be. Look for low-cost bulk items, or an even better option is to ask parents for snack donations to the classroom.

Example activity: Create a graph

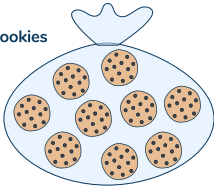
Give students a small fun-size bag of multi-colored candy or food such as M&Ms, Skittles, or you could even use the marshmallows from a box of Lucky Charms!

For this activity, students sort out their group of 'manipulatives' into colors. Then, students will create a graph outline (or you can print one out for them and they can label them.) This will work best as a bar graph or pictograph. Students will plot their 'data' on the graph and then answer questions such as, *which color do you have the most of?*, *which do you have the least of?*, *how many more yellow do you have than blue?*, etc. For younger students, you can have them place the food right on their paper to create the graph, while older students can color in the graph to represent it.

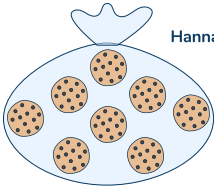
Go further

Pete and Hannah each have a bag of cookies.

Pete's cookies



Hannah's cookies



a Who has an even number of cookies in their bag?

b If Pete and Hannah combine their cookies, will they have an even or odd number of cookies?

c If each bag had an odd number of cookies and we added them together, would the total number of cookies be odd or even? How do you know?

Grade 2 - Even and odd numbers using visuals

Here we use food to help pupils understand odd and even numbers in a real life context. They can see that when they pair the numbers, Pete has one cookie left over so he must have an odd number of cookies.

Do it yourself

You could make your own geoboards using corkboard and pins, or using a flat piece of wood and screws with rubber bands.

Example activity: Perimeter

Instruct children to use the rubber bands to make a shape with a given number and length of sides. They can work out the perimeter by counting all the points around the shape.

Let's go through this together

What do you know about the properties of triangles?

Two of the angles in a triangle are 40° and 65°

Trixie says: "The triangle is isosceles."

Explain why Trixie is not correct.

What do the angles in every triangle total? °

.....

What is the total of two of the angles in Trixie's triangle?

° + ° = °

.....

What will be the 3rd angle of Trixie's triangle?

° 0 ° = °

.....

Draw an isosceles triangle.

What is special about the angles in an isosceles triangle?

Grade 4 - Reasoning with Angles

Here we use isometric paper to mimic a geoboard. In the classroom you could ask students to make a range of different triangles on a geoboard with rubber bands and then sort them into different types of triangles based on their properties.

Extra reading and support

[Didax Virtual Manipulatives](#)

[Mathsbot](#)

[How Math Manipulatives Transform Elementary Math Lessons - Third Space Learning Blog](#)

[Concrete Resources Explained - TSL Blog](#)

[What is The Concrete Representational Abstract \(CRA\) Approach - Third Space Learning Blog](#)

References

¹ <https://www.didax.com/apps/base-ten-blocks/>
<https://oryxlearning.com/manipulatives/base-ten-blocks>

² <https://mathsbot.com/manipulatives/placeValueCounters>

³ <https://www.didax.com/apps/two-color-counters/>

⁴ <https://www.didax.com/apps/unifix/>

⁵ <https://www.didax.com/apps/120-board/>

⁶ <https://www.didax.com/apps/ten-frame/>

⁷ <https://www.didax.com/apps/pattern-blocks/>

⁸ <https://www.didax.com/apps/fraction-number-line/>

⁹ <https://toytheater.com/fraction-circles/>

¹⁰ <https://mathsbot.com/manipulatives/unitbox>

¹¹ <https://www.didax.com/apps/geoboard/>

¹² <https://deck.of.cards/>

¹³ <https://freeonlinedice.com/>

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