Recurrence Relation - Worksheet

Skill

Group A - Generating a sequence from a recurrence relation

For each of the following find U_2, U_3 and U_4:

1) $U_{n+1} = 10U_n$, $U_1 = 4$

2) $U_{n+1} = 10U_n + 3$, $U_1 = 4$

3) $U_{n+1} = 10U_n - 3$, $U_1 = 4$

4) $U_{n+1} = 5U_n + 3$, $U_1 = 2$

5) $U_{n+1} = 5U_n + 3$, $U_1 = -2$

6) $U_{n+1} = 5U_n + 3$, $U_1 = 0$

7) $U_{n+1} = U_n^2 - 2U_n$, $U_1 = 1$

8) $U_{n+1} = 2U_n - U_n^2$, $U_1 = 1$

9) $U_{n+1} = 2U_n + U_n^2$, $U_1 = 1$

10) $U_{n+1} = 2 - \frac{4}{U_n}$, $U_1 = 4$

11) $U_{n+1} = \frac{4}{U_n} + 2$, $U_1 = 4$

12) $U_{n+1} = \frac{4}{U_n} - 2$, $U_1 = 4$

Group B - Describing a sequence using recurrence relations

Describe the following sequences using a recurrence relation:

1) 40, 50, 60, 70, ...

2) 0, −3, −6, −9, ...

3) −1, 3, 7, 11, ...

4) 44, 40, 36, 32, ...

5) 60, 58, 56, 54, ...

6) 87, 92, 97, 102, ...

7) 5, 10, 20, 40, ...

8) 3, 9, 27, 81, 243, ...

9) 1, 4, 16, 64, ...

10) 500, 250, 125, 62.5, ...

11) 180, 60, 20, ...

12) 8, 2, $\frac{1}{2}$, $\frac{1}{8}$, ...

Recurrence Relation - Worksheet

Group C - Solving linear recurrence relations

If the recurrence relation is in the form $U_{n+1} = aU_n + b$.

Find the values of the constants a and b, when:

1) $U_1 = 1,\ U_2 = 3,\ U_3 = 5$

2) $U_1 = 2,\ U_2 = -2,\ U_3 = -6$

3) $U_1 = 1,\ U_2 = 7,\ U_3 = 13$

4) $U_1 = 1,\ U_2 = 3,\ U_3 = 7$

5) $U_1 = 1,\ U_2 = -2,\ U_3 = -8$

6) $U_1 = 1,\ U_2 = -1,\ U_3 = 7$

7) $U_1 = 1,\ U_2 = -1,\ U_3 = 9$

8) $U_1 = 2,\ U_2 = 8,\ U_3 = 38$

9) $U_1 = 2,\ U_2 = 18,\ U_3 = 130$

10) $U_1 = 1,\ U_2 = 4,\ U_3 = -5$

11) $U_1 = 1,\ U_2 = 3,\ U_3 = -1$

12) $U_1 = 1,\ U_2 = 4,\ U_3 = 16$

Recurrence Relation - Worksheet

Applied

1) A sequence is generated by a recurrence relation of the form $U_{n+1} = kU_n + 3$, where k is constant. If $U_1 = 4$ and $U_2 = 11$.

Find the values of k, U_3 and U_4.

2) A sequence is generated by the recurrence relation $U_{n+1} = 3U_n + c$, $U_1 = 2$.

Given that $U_4 = 28$, find c.

3) For each recurrence relation calculate the value of U_5.

(a) $U_{n+1} = 3U_n + 1,\ U_0 = 2$

(b) $U_{n+1} = -4U_n + 3,\ U_0 = 1$

(c) $U_n = 0.5U_{n-1} + 2,\ U_0 = 12$

4) For the sequence defined by the recurrence relation $U_{n+1} = 2U_n - 1$ and $U_0 = 3$.

(a) Find the values of U_1, U_2, U_3 and U_9

(b) Find which term of the sequence is the first to exceed 50 000.

5) A recurrence relation is given by $U_{n+1} = 0.3U_n - 4,\ U_0 = 8$.

(a) Calculate the values of U_2 and U_3.

(b) Find the smallest value of n such that $U_n < -5.7$.

Recurrence Relation - Exam Questions

1) A sequence U_1, U_2, U_3, U_4, ... is given by the recurrence relation

$U_{n+1} = 3U_n + 2$

$U_1 = 2$

Find the value of U_2, U_3, U_4 and U_5.

.....

(4 marks)

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2) A sequence is defined by the recurrence relation

$U_n = 0.9U_{n-1} + 2$

$U_1 = 3$

(a) Calculate the value of U_2.

.....

(2)

(b) What is the smallest value of n for which $U_n > 10$?

.....

(2)

(4 marks)

Recurrence Relation - Exam Questions

3) Find the first 5 terms of the sequence in which $U_1 = 7$ and $U_{n+1} = U_n + n$ for $n \geq 1$.

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(2 marks)

Recurrence Relation - Answers

	Question	Answer
	Skill Questions	
Group A	For each of the following find U_2, U_3 and U_4:	
	1) $U_{n+1} = 10U_n$, $U_1 = 4$	**1)** $40, 400, 4000$
	2) $U_{n+1} = 10U_n + 3$, $U_1 = 4$	**2)** $43, 433, 4333$
	3) $U_{n+1} = 10U_n - 3$, $U_1 = 4$	**3)** $37, 367, 3667$
	4) $U_{n+1} = 5U_n + 3$, $U_1 = 2$	**4)** $13, 68, 343$
	5) $U_{n+1} = 5U_n + 3$, $U_1 = -2$	**5)** $-7, -32, -157$
	6) $U_{n+1} = 5U_n + 3$, $U_1 = 0$	**6)** $3, 18, 93$
	7) $U_{n+1} = U_n^2 - 2U_n$, $U_1 = 1$	**7)** $-1, 3, 3$
	8) $U_{n+1} = 2U_n - U_n^2$, $U_1 = 1$	**8)** $1, 1, 1$
	9) $U_{n+1} = 2U_n + U_n^2$, $U_1 = 1$	**9)** $3, 15, 255$
	10) $U_{n+1} = 2 - \frac{4}{U_n}$, $U_1 = 4$	**10)** $1, -2, 4$
	11) $U_{n+1} = \frac{4}{U_n} + 2$, $U_1 = 4$	**11)** $3, \frac{10}{3}, \frac{16}{5}$
	12) $U_{n+1} = \frac{4}{U_n} - 2$, $U_1 = 4$	**12)** $-1, -6, -\frac{8}{3}$

Recurrence Relation - Answers

Group B	Describe the following sequences using a recurrence relation:	
	1) $40,\ 50,\ 60,\ 70,\ ...$	**1)** $U_{n+1} = U_n + 10,\ U_1 = 40$
	2) $0,\ -3,\ -6,\ -9,\ ...$	**2)** $U_{n+1} = U_n - 3,\ U_1 = 0$
	3) $-1,\ 3,\ 7,\ 11,\ ...$	**3)** $U_{n+1} = U_n + 4,\ U_1 = -1$
	4) $44,\ 40,\ 36,\ 32,\ ...$	**4)** $U_{n+1} = U_n - 4,\ U_1 = 44$
	5) $60,\ 58,\ 56,\ 54,\ ...$	**5)** $U_{n+1} = U_n - 2,\ U_1 = 60$
	6) $87,\ 92,\ 97,\ 102,\ ...$	**6)** $U_{n+1} = U_n + 5,\ U_1 = 87$
	7) $5,\ 10,\ 20,\ 40,\ ...$	**7)** $U_{n+1} = 2U_n,\ U_1 = 5$
	8) $3,\ 9,\ 27,\ 81,\ 243,\ ...$	**8)** $U_{n+1} = 3U_n,\ U_1 = 3$
	9) $1,\ 4,\ 16,\ 64,\ ...$	**9)** $U_{n+1} = 4U_n,\ U_1 = 1$
	10) $500,\ 250,\ 125,\ 62.5,\ ...$	**10)** $U_{n+1} = \frac{U_n}{2},\ U_1 = 500$
	11) $180,\ 60,\ 20,\ ...$	**11)** $U_{n+1} = \frac{U_n}{3},\ U_1 = 180$
	12) $8,\ 2,\ \frac{1}{2},\ \frac{1}{8},\ ...$	**12)** $U_{n+1} = \frac{U_n}{4},\ U_1 = 8$

Recurrence Relation - Answers

Group C	If the recurrence relation is in the form $U_{n+1} = aU_n + b$. Find the values of the constants a and b, when:	
	1) $U_1 = 1, U_2 = 3, U_3 = 5$	**1)** $a = 1, b = 2$
	2) $U_1 = 2, U_2 = -2, U_3 = -6$	**2)** $a = 1, b = -4$
	3) $U_1 = 1, U_2 = 7, U_3 = 13$	**3)** $a = 1, b = 6$
	4) $U_1 = 1, U_2 = 3, U_3 = 7$	**4)** $a = 2, b = 1$
	5) $U_1 = 1, U_2 = -2, U_3 = -8$	**5)** $a = 2, b = -4$
	6) $U_1 = 1, U_2 = -1, U_3 = 7$	**6)** $a = -4, b = 3$
	7) $U_1 = 1, U_2 = -1, U_3 = 9$	**7)** $a = -5, b = 4$
	8) $U_1 = 2, U_2 = 8, U_3 = 38$	**8)** $a = 5, b = -2$
	9) $U_1 = 2, U_2 = 18, U_3 = 130$	**9)** $a = 7, b = 4$
	10) $U_1 = 1, U_2 = 4, U_3 = -5$	**10)** $a = -3, b = 7$
	11) $U_1 = 1, U_2 = 3, U_3 = -1$	**11)** $a = -2, b = 5$
	12) $U_1 = 1, U_2 = 4, U_3 = 16$	**12)** $a = 4, b = 0$

Recurrence Relation - Answers

	Question	Answer
	Applied Questions	
1)	A sequence is generated by a recurrence relation of the form $U_{n+1} = kU_n + 3$, where k is constant. If $U_1 = 4$ and $U_2 = 11$. Find the values of k, U_3 and U_4.	$U_2 = kU_1 + 3$ $11 = 4k + 3$ $k = 2$ $U_3 = 25$ $U_4 = 53$
2)	A sequence is generated by the recurrence relation $U_{n+1} = 3U_n + c$, $U_1 = 2$. Given that $U_4 = 28$, find c.	$U_2 = 3U_1 + c = 6 + c$ $U_3 = 3(6 + c) + c = 18 + 4c$ $U_4 = 3(18 + 4c) + c$ $28 = 54 + 13c$ $c = -2$
3)	For each recurrence relation calculate the value of U_5 **a)** $U_{n+1} = 3U_n + 1$, $U_0 = 2$ **b)** $U_{n+1} = -4U_n + 3$, $U_0 = 1$ **c)** $U_n = 0.5U_{n-1} + 2$, $U_0 = 12$	**a)** 607 **b)** -409 **c)** 4.25
4)	For the sequence defined by the recurrence relation $U_{n+1} = 2U_n - 1$ and $U_0 = 3$ **a)** Find the values of U_1, U_2, U_3 and U_9 **b)** Find which term of the sequence is the first to exceed 50 000	**a)** $U_1 = 5$, $U_2 = 9$ $U_3 = 17$, $U_9 = 1025$ **b)** U_{15}
5)	A recurrence relation is given by $U_{n+1} = 0.3U_n - 4$, $U_0 = 8$ **a)** Calculate the values of U_2 and U_3 **b)** Find the smallest value of n such that $U_n < -5.7$	**a)** $U_2 = -4.48$, $U_3 = -5.344$ **b)** $n = 6$

Recurrence Relation - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	A sequence U_1, U_2, U_3, U_4, ... is given by the recurrence relation $U_{n+1} = 3U_n + 2$ $U_1 = 2$ Find the value of U_2, U_3, U_4 and U_5.	$U_2 = 3(2) + 2 = 8$ $U_3 = 3(8) + 2 = 26$ $U_4 = 3(26) + 2 = 80$ $U_5 = 3(80) + 2 = 242$	**(1)** **(1)** **(1)** **(1)**
2)	A sequence is defined by the recurrence relation $U_n = 0.9U_{n-1} + 2$ $U_1 = 3$		
(a)	Calculate the value of U_2	**(a)** $(0.9 \times 3) + 2$ 4.7	**(1)** **(1)**
(b)	What is the smallest value of n for which $U_n > 10$?	**(b)** $U_3 = 6.23$ $U_4 = 7.607$ $U_5 = 8.8463$ $U_6 = 9.96167$ $U_7 = 10.965503$ 7	**(1)** **(1)** **(1)**
3)	Find the first 5 terms of the sequence in which $U_1 = 7$ and $U_{n+1} = U_n + n$ for $n \geq 1$.	$U_2 = 7 + 1 = 8$ $U_3 = 8 + 2 = 10$ $U_4 = 10 + 3 = 13$ $U_5 = 13 + 4 = 17$ 8, 10, 13, 17, ...	**(1)** **(1)** **(1)**