Functions in Algebra - Worksheet

Skill

Group A - Solving equations using function machines

Draw a function machine to represent the equation and use it to solve the equation.

1) $x + 4 = 9$

2) $5x = 35$

3) $\frac{x}{9} = 3$

4) $3x + 4 = 22$

5) $2x - 1 = 14$

6) $6(x + 7) = 18$

7) $\frac{x}{4} - 3 = 4$

8) $\frac{x}{5} + 7 = 2$

9) $\frac{x-2}{3} = 8$

Group B - Evaluating and finding composite functions

If $f(x) = x - 2$, $g(x) = x^2$ and $h(x) = 5x$, evaluate or find the expressions for these composite functions.

1) $fg(1)$

2) $gf(3)$

3) $hf(2)$

4) $fhg(-4)$

5) $gh(x)$

6) $hg(x)$

7) $gf(x)$

8) $fhg(x)$

9) $gfh(x)$

Group C - Finding inverse functions

Find the inverse of these functions.

1) $f^{-1}(x)$ when
   $f(x) = x - 2$

2) $g^{-1}(x)$ when
   $g(x) = 6x$

3) $h^{-1}(x)$ when
   $h(x) = 3x + 8$

4) $f^{-1}(x)$ when
   $f(x) = \frac{x}{5} + 1$

5) $g^{-1}(x)$ when
   $g(x) = \frac{5x}{3} - 2$

6) $h^{-1}(x)$ when
   $h(x) = \frac{2(x+1)}{3}$

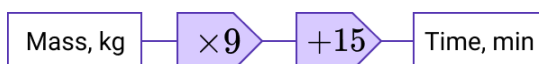**7)** $f^{-1}(x)$ when
   $f(x) = \sqrt{x} - 1$

8) $g^{-1}(x)$ when
   $g(x) = \sqrt[3]{x + 5}$

9) $h^{-1}(x)$ when
   $h(x) = 3x^3 - 4$

Functions in Algebra - Worksheet

Applied

1) The length of time, in minutes, required to cook a joint of meat, can be calculated using the funchine machine.

| Mass, kg | $\times 9$ | $+15$ | Time, min |

(a) Find the time required to cook a joint of meat with mass $4kg$.

(b) A joint of meat was cooked for 37.5 minutes. What was its mass?

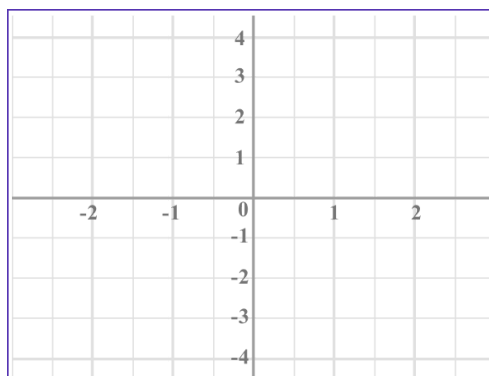**2)** **(a)** Draw a function machine for the function $g(x) = 2x^2 + 7$.

(b) Draw a function machine for $h(x)$ if $h^{-1}(x) = \sqrt{\frac{x}{3} + 1}$.

3) $f(x) = \frac{1}{2}x + 2$

(a) Fill in the table for $f(x)$

x	-2	-1	0	1	2
$f(x)$					

(b) Plot the graph of $f(x)$ for values of x between -2 and 2.

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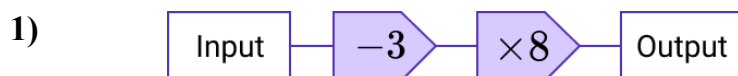
2

Functions in Algebra - Worksheet

4)   If $g(x) = \sqrt{x} + 4$ and $f(x) = 3x - 10$

(a) Find $ff(a + 2)$.

(b) Find values of x that satisfy $g^{-1}(x) = f(x)$.
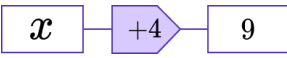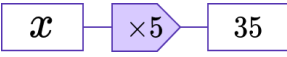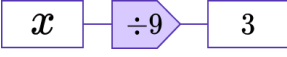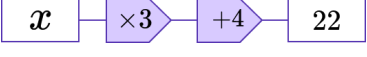
Functions in Algebra - Exam Questions

1)

| Input |—| -3 |—| $\times 8$ |—| Output |

    (a) Find the output if the input is 10.

    ....

    (1)

    (b) Write a simplified expression for the input if the output is $24n - 32$.

    ....

    (3)

    (4 marks)

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2)   If $f(x) = 5 - 2x$ and $g(x) = 2x + 6$

    (a) Find when $f(x) = -9$.

    ....

    (2)

    (b) Write an expression for $gf(x)$.

    ....

    (3)

    (5 marks)

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3) $g(x) = \dfrac{x-3}{4}$ and $h(x) = \dfrac{x}{2} + 3$

    Solve when $g^{-1}(x) = h^{-1}(x)$.

    ....

    (4 marks)

---

4) $f(x) = x^2 + 1$ and $g(x) = 3x - 1$

    (a) Find $fg(-8)$.

    ....

    (2)

    (b) Find values of x that satisfy $gf(x) = 77$.

    ....

    (4)

    (6 marks)

Functions in Algebra - Answers

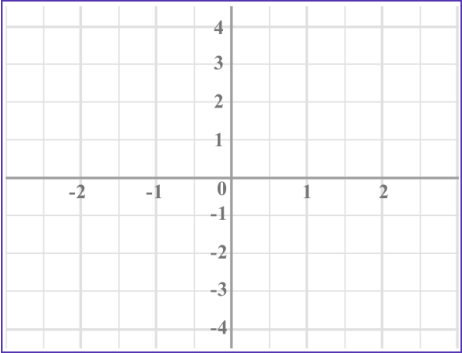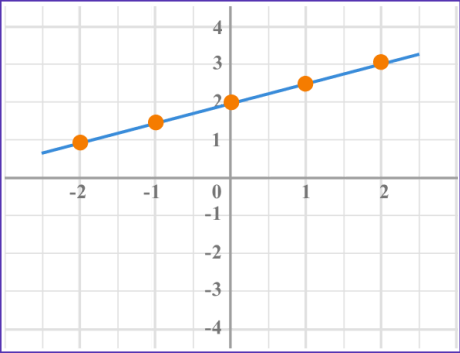|	Question	Answer
	Skill Questions	
Group A	Draw a function machine to represent the equation and use it to solve the equation. **1)** $x + 4 = 9$ **2)** $5x = 35$ **3)** $\frac{x}{9} = 3$ **4)** $3x + 4 = 22$ **5)** $2x - 1 = 14$ **6)** $6(x + 7) = 18$ **7)** $\frac{x}{4} - 3 = 4$ **8)** $\frac{x}{5} + 7 = 2$ **9)** $\frac{x-2}{3} = 8$	**1)** $\boxed{x}$ — $\boxed{+4}$ — $\boxed{9}$ $x = 5$ **2)** $\boxed{x}$ — $\boxed{\times 5}$ — $\boxed{35}$ $x = 7$ **3)** $\boxed{x}$ — $\boxed{\div 9}$ — $\boxed{3}$ $x = 27$ **4)** $\boxed{x}$ — $\boxed{\times 3}$ — $\boxed{+4}$ — $\boxed{22}$ $x = 6$ **5)** $\boxed{x}$ — $\boxed{\times 2}$ — $\boxed{-1}$ — $\boxed{14}$ $x = 7.5$ **6)** $\boxed{x}$ — $\boxed{+7}$ — $\boxed{\times 6}$ — $\boxed{18}$ $x = -4$ **7)** $\boxed{x}$ — $\boxed{\div 4}$ — $\boxed{-3}$ — $\boxed{4}$ $x = 28$ **8)** $\boxed{x}$ — $\boxed{\div 5}$ — $\boxed{+7}$ — $\boxed{2}$ $x = -25$ **9)** $\boxed{x}$ — $\boxed{-2}$ — $\boxed{\div 3}$ — $\boxed{8}$ $x = 26$
Group B	If $f(x) = x - 2$, $g(x) = x^2$ and $h(x) = 5x$, evaluate or find the expressions for these composite functions. **1)** $fg(1)$	**1)** -1

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Functions in Algebra - Answers

	2) $gf(3)$ **3)** $hf(5)$ **4)** $fhg(-4)$ **5)** $gh(x)$ **6)** $hg(x)$ **7)** $gf(x)$ **8)** $fhg(x)$ **9)** $gfh(x)$	**2)** 1 **3)** 15 **4)** 78 **5)** $25x^2$ **6)** $5x^2$ **7)** $x^2 - 4x + 4$ **8)** $5x^2 - 2$ **9)** $25x^2 - 20x$
Group C	Find the inverse of these functions. **1)** $f^{-1}(x)$ when $f(x) = x - 2$ **2)** $g^{-1}(x)$ when $g(x) = 6x$ **3)** $h^{-1}(x)$ when $h(x) = 3x + 8$ **4)** $f^{-1}(x)$ when $f(x) = \dfrac{x}{5} + 1$ **5)** $g^{-1}(x)$ when $g(x) = \dfrac{5x}{3} - 2$ **6)** $h^{-1}(x)$ when $h(x) = \dfrac{2(x+1)}{3}$ **7)** $f^{-1}(x)$ when $f(x) = \sqrt{x} - 1$ **8)** $g^{-1}(x)$ when $g(x) = \sqrt[3]{x + 5}$ **9)** $h^{-1}(x)$ when $h(x) = 3x^3 - 4$	**1)** $x + 2$ **2)** $\dfrac{x}{6}$ **3)** $\dfrac{x-8}{3}$ **4)** $5x - 5$ **5)** $\dfrac{3x+6}{5}$ **6)** $\dfrac{3x}{2} - 1$ **7)** $x^2 + 2x + 1$ **8)** $x^3 - 5$ **9)** $\sqrt[3]{\dfrac{x+4}{3}}$

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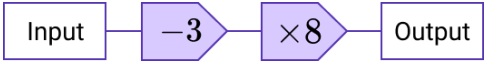
Functions in Algebra - Answers

	Question	Answer
	Applied Questions	
1)	The length of time, in minutes, required to cook a joint of meat, can be calculated using the funchine machine. Mass, kg — $\times 9$ — $+15$ — Time, min **(a)** Find the time required to cook a joint of meat with mass $4kg$. **(b)** A joint of meat was cooked for 37.5 minutes. What was its mass?	**(a)** 51 minutes **(b)** $2.5kg$
2)	**(a)** Draw a function machine for the function $g(x) = 2x^2 + 7$. **(b)** Draw a function machine for $h(x)$ if $h^{-1}(x) = \sqrt{\dfrac{x}{3} + 1}$.	**(a)** x — square — $\times 2$ — $+7$ — $g(x)$ **(b)** x — square — -1 — $\times 3$ — $h(x)$
3)	$f(x) = \dfrac{1}{2}x + 2$ Fill in the table for $f(x)$ **(a)** <table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x)$</td><td></td><td></td><td></td><td></td><td></td></tr></table>	**(a)** <table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x)$</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr></table>

Functions in Algebra - Answers

<table>
<tr>
<td></td>
<td>(b) Plot the graph of $f(x)$ for values of x between -2 and 2.

</td>
<td>(b)
</td>
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<tr>
<td>4)</td>
<td colspan="2">If $g(x) = \sqrt{x} + 4$ and $f(x) = 3x - 10$</td>
</tr>
<tr>
<td></td>
<td>(a) Find $ff(a + 2)$.</td>
<td>(a) $3(3(a + 2) - 10) - 10$
$9a - 22$</td>
</tr>
<tr>
<td></td>
<td>(b) Find values of x that satisfy $g^{-1}(x) = f(x)$.</td>
<td>(b) $g^{-1}(x) = (x - 4)^2$
$(x - 4)^2 = 3x - 10$
$x^2 - 5x + 6 = 0$
$x = 2, 3$</td>
</tr>
</table>

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Functions in Algebra - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	Input —[−3]—[×8]— Output		
(a)	Find the output if the input is 10.	**(a)** 56	**(1)**
(b)	Write a simplified expression for the input if the output is $24n - 32$	**(b)** $\dfrac{24n-32}{8}$ $3n - 4 + 3$ $3n - 1$	**(3)**
2)	If $f(x) = 5 - 2x$ and $g(x) = 2x + 6$		
(a)	Find when $f(x) = -9$	**(a)** Sight of $2x = 14$ $x = 7$	**(2)**
(b)	Write an expression for $gf(x)$	**(b)** $2(5 - 2x) + 6$ $10 - 4x + 6$ $16 - 4x$	**(3)**
3)	$g(x) = \dfrac{x-3}{4}$ and $h(x) = \dfrac{x}{2} + 3$		
	Solve when $g^{-1}(x) = h^{-1}(x)$	$g^{-1}(x) = 4x + 3$ $h^{-1}(x) = 2(x - 3)$ or $2x - 6$ Set equal $4x + 3 = 2x - 6$ Answer $x = -4.5$	**(4)**
4)	$f(x) = x^2 + 1$ and $g(x) = 3x - 1$		
(a)	Find $fg(-8)$	**(a)** $g(-8) = -25$ $f(-25) = 626$	**(2)**

Functions in Algebra - Mark Scheme

(b)	Find values of x that satisfy $gf(x) = 77$	**(b)** $3(x^2 + 1) - 1$ seen $3x^2 + 2 = 77$ $x^2 = 25$ $x = 5, -5$ both answers	**(4)**

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