

## Upper and Lower Bounds - Worksheet

### Skill

#### Group A - Finding upper and lower bounds

Find the upper and lower bounds of the rounded values.

- |                                  |                                   |                                 |
|----------------------------------|-----------------------------------|---------------------------------|
| 1) 12 to the nearest integer     | 2) 8.7 to one decimal place       | 3) 300 to the nearest 100       |
| 4) 300 to the nearest 10         | 5) 5000 to 1 significant figure   | 6) 5000 to 2 significant figure |
| 7) 1.78 to 3 significant figures | 8) 0.030 to 3 decimal places      | 9) 4670 to the nearest integer  |
| 10) 4670 to the nearest 10       | 11) 0.62 to 2 significant figures | 12) 60.0 to one decimal place   |

#### Group B - Bound calculations involving addition or multiplication

If  $A = 7$  to the nearest integer,  $B = 3.2$  to 2 significant figures,  $C = 40$  to the nearest 10 and  $D = 12.39$  to 2 decimal places. Find the upper and lower bounds of the calculations.

- |            |               |              |
|------------|---------------|--------------|
| 1) $A + B$ | 2) $A + C$    | 3) $B + D$   |
| 4) $AB$    | 5) $BD$       | 6) $CB$      |
| 7) $2A$    | 8) $2(B + C)$ | 9) $2B + 3D$ |
| 10) $A^2$  | 11) $B^3$     | 12) $ABD$    |

#### Group C - Bound calculations involving subtraction or division

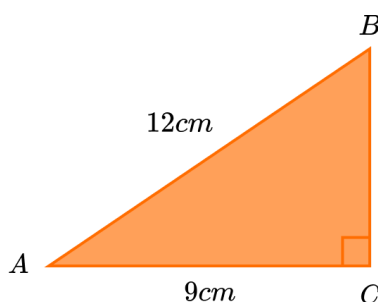
If  $P = 20$  to the nearest integer,  $Q = 100$  to the nearest hundred,  $R = 5.2$  to 1 decimal place and  $S = 320$  to 2 significant figures. Find the upper and lower bounds of the calculations.

- |                  |                    |                    |
|------------------|--------------------|--------------------|
| 1) $S - Q$       | 2) $Q - P$         | 3) $P - R$         |
| 4) $\frac{Q}{P}$ | 5) $\frac{P}{Q}$   | 6) $\frac{S}{P}$   |
| 7) $\frac{R}{S}$ | 8) $P - 2R$        | 9) $4Q - S$        |
| 10) $S - P - Q$  | 11) $\frac{QP}{S}$ | 12) $\frac{P}{RQ}$ |

## Upper and Lower Bounds - Worksheet

### Applied

- 1) A rectangle has a length of  $7.2\text{cm}$  and a width of  $4.8\text{cm}$ . Both measurements are to the nearest millimetre.
- (a) Find the upper and lower bounds of the perimeter of the rectangle.
  - (b) Find the upper and lower bounds of the area of the rectangle.
- 2) A circular pond has a diameter of  $5.36\text{m}$  to the nearest centimetre.
- (a) Find the upper bound of the circumference of the pond. Give your answer to 3 decimal places.
  - (b) Find the lower bound of the area of the pond. Give your answer to 3 decimal places.
- 3) The diagram shows the right-angled triangle  $ABC$  where  $AB = 12\text{cm}$  and  $AC = 9\text{cm}$ . Both measurements have been rounded to the nearest centimetre.



- (a) Find the lower bound for the length  $BC$ .
- (b) Find the error interval for the angle  $BAC$ .

## Upper and Lower Bounds - Worksheet

**4)**

Using the formula  $P = \frac{F}{A}$ .

- (a)** Find the upper bound of  $P$  if  $F = 45.72$  to 2 d.p and  $A = 5.8$  to 1 d.p.
- (b)** If  $F = 5.21$  to 2 d.p and  $P = 1.67$  to 2 d.p. By considering bounds, find the value of  $A$  to a suitable degree of accuracy.

## Upper and Lower Bounds - Exam Questions

- 1)  $E = mgh$   
 $m = 4$  correct to the nearest integer.  
 $g = 9.81$  correct to 3 significant figures.  
 $h = 2.4$  correct to 1 decimal place.

- (a) Find the lower bound for  $E$ .

.....  
(2)

- (b) Find the upper bound for  $E$ .

.....  
(2)  
(4 marks)

- 
- 2) A rectangular field has a length of 260 metres to 2 significant figures and a width of 145 metres to the nearest metre.

- (a) Find the lower bound of the area of the field.

.....  
(2)

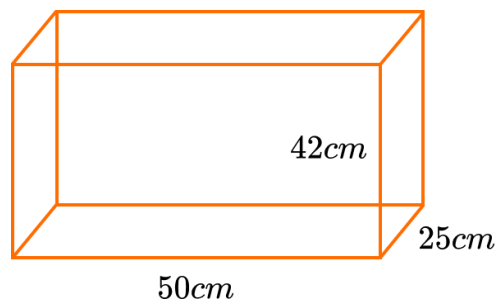
- (b) Find the upper bound of the perimeter of the field.

.....  
(2)  
(4 marks)

## Upper and Lower Bounds - Exam Questions

- 3) A container in the shape of a cuboid is to be filled with soil. The cuboid is  $50\text{cm}$  by  $42\text{cm}$  by  $25\text{cm}$ , where the measurements are correct to the nearest cm.

The container will be filled using a jug that can hold  $2500\text{ml}$  to the nearest  $100\text{ ml}$ .



What is the minimum number of full jugs that may be required to fill the container?

.....  
(5 marks)

4)  $K = \sqrt{\frac{M}{H}}$

$M = 8.42$  to 3 significant figures

$H = 2.64$  to 3 significant figures.

By considering bounds, work out the value of  $K$  to a suitable degree of accuracy, justifying your answer.

.....  
(5 marks)

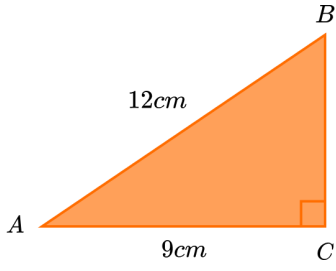
## Upper and Lower Bounds - Answers

	Question	Answer
	Skill Questions	
Group A	<p>Find the upper and lower bounds of the rounded values.</p> <p><b>1)</b> 12 to the nearest integer  <b>2)</b> 8.7 to one decimal place  <b>3)</b> 300 to the nearest 100  <b>4)</b> 300 to the nearest 10  <b>5)</b> 5000 to 1 significant figure  <b>6)</b> 5000 to 2 significant figure  <b>7)</b> 1.78 to 3 significant figures  <b>8)</b> 0.030 to 3 decimal places  <b>9)</b> 4670 to the nearest integer  <b>10)</b> 4670 to the nearest 10  <b>11)</b> 0.62 to 2 significant figures  <b>12)</b> 60.0 to one decimal place</p>	<p><b>1)</b> <math>UB = 12.5</math>, <math>LB = 11.5</math>  <b>2)</b> <math>UB = 8.75</math>, <math>LB = 8.65</math>  <b>3)</b> <math>UB = 350</math>, <math>LB = 250</math>  <b>4)</b> <math>UB = 305</math>, <math>LB = 295</math>  <b>5)</b> <math>UB = 5500</math>, <math>LB = 4500</math>  <b>6)</b> <math>UB = 5050</math>, <math>LB = 4950</math>  <b>7)</b> <math>UB = 1.785</math>, <math>LB = 1.775</math>  <b>8)</b> <math>UB = 0.0305</math>, <math>LB = 0.0295</math>  <b>9)</b> <math>UB = 4670.5</math>, <math>LB = 4669.5</math>  <b>10)</b> <math>UB = 4675</math>, <math>LB = 4665</math>  <b>11)</b> <math>UB = 0.625</math>, <math>LB = 0.615</math>  <b>12)</b> <math>UB = 60.05</math>, <math>LB = 59.95</math></p>
Group B	<p>If <math>A = 7</math> to the nearest integer, <math>B = 3.2</math> to 2 significant figures, <math>C = 40</math> to the nearest 10 and <math>D = 12.39</math> to 2 decimal places. Find the upper and lower bounds of the calculations.</p> <p><b>1)</b> <math>A + B</math>  <b>2)</b> <math>A + C</math>  <b>3)</b> <math>B + D</math>  <b>4)</b> <math>AB</math>  <b>5)</b> <math>BD</math>  <b>6)</b> <math>CB</math>  <b>7)</b> <math>2A</math>  <b>8)</b> <math>2(B + C)</math>  <b>9)</b> <math>2B + 3D</math></p>	<p><math>UB_A = 7.5</math>, <math>LB_A = 6.5</math>  <math>UB_B = 3.25</math>, <math>LB_B = 3.15</math>  <math>UB_C = 45</math>, <math>LB_C = 35</math>  <math>UB_D = 12.395</math>, <math>LB_D = 12.385</math></p> <p><b>1)</b> <math>UB = 10.75</math>, <math>LB = 9.65</math>  <b>2)</b> <math>UB = 52.5</math>, <math>LB = 41.5</math>  <b>3)</b> <math>UB = 15.645</math>, <math>LB = 15.535</math>  <b>4)</b> <math>UB = 24.375</math>, <math>LB = 20.475</math>  <b>5)</b> <math>UB = 40.28375</math>, <math>LB = 39.01275</math>  <b>6)</b> <math>UB = 146.25</math>, <math>LB = 110.25</math>  <b>7)</b> <math>UB = 15</math>, <math>LB = 13</math>  <b>8)</b> <math>UB = 96.5</math>, <math>LB = 76.3</math>  <b>9)</b> <math>UB = 43.685</math>, <math>LB = 43.455</math></p>

## Upper and Lower Bounds - Answers

Group B contd	<b>10)</b> $A^2$ <b>11)</b> $B^3$ <b>12)</b> $ABD$	<b>10)</b> $UB = 56.25, \quad LB = 42.25$ <b>11)</b> $UB = 34.328125, \quad LB = 31.255875$ <b>12)</b> $UB = 302.128125, \quad LB = 253.582875$
Group C	<p>If <math>P = 20</math> to the nearest integer,  <math>Q = 100</math> to the nearest hundred,  <math>R = 5.2</math> to 1 decimal place and  <math>S = 320</math> to 2 significant figures.  Find the upper and lower bounds  of the calculations.</p> <p><b>1)</b> <math>S - Q</math>  <b>2)</b> <math>Q - P</math>  <b>3)</b> <math>P - R</math>  <b>4)</b> <math>\frac{Q}{P}</math>  <b>5)</b> <math>\frac{P}{Q}</math>  <b>6)</b> <math>\frac{S}{P}</math>  <b>7)</b> <math>\frac{R}{S}</math>  <b>8)</b> <math>P - 2R</math>  <b>9)</b> <math>4Q - S</math>  <b>10)</b> <math>S - P - Q</math>  <b>11)</b> <math>\frac{QP}{S}</math>  <b>12)</b> <math>\frac{P}{RQ}</math></p>	$UB_P = 20.5, \quad LB_P = 19.5$ $UB_Q = 150, \quad LB_Q = 50$ $UB_R = 5.25, \quad LB_R = 5.15$ $UB_S = 325, \quad LB_S = 315$  <b>1)</b> $UB = 275, \quad LB = 165$ <b>2)</b> $UB = 130.5, \quad LB = 29.5$ <b>3)</b> $UB = 15.35, \quad LB = 14.25$ <b>4)</b> $UB = 7.6923, \quad LB = 2.4390$ <b>5)</b> $UB = 0.41, \quad LB = 0.13$ <b>6)</b> $UB = 16.\dot{6}, \quad LB = 15.366$ <b>7)</b> $UB = 0.01\dot{6}, \quad LB = 0.0158$ <b>8)</b> $UB = 10.2, \quad LB = 9$ <b>9)</b> $UB = 285, \quad LB = -125$ <b>10)</b> $UB = 255.5, \quad LB = 144.5$ <b>11)</b> $UB = 9.7619, \quad LB = 3$ <b>12)</b> $UB = 0.0796, \quad LB = 0.0248$

## Upper and Lower Bounds - Answers

	Question	Answer
	Applied Questions	
1)	<p>A rectangle has a length of <math>7.2\text{ cm}</math> and a width of <math>4.8\text{ cm}</math>. Both measurements are to the nearest millimetre.</p> <p>a) Find the upper and lower bounds of the perimeter of the rectangle.</p> <p>b) Find the upper and lower bounds of the area of the rectangle.</p>	<p>a) <math>UB = 24.2\text{ cm}</math> <math>LB = 23.8\text{ cm}</math></p> <p>b) <math>UB = 35.1625\text{ cm}^2</math> <math>LB = 33.9625\text{ cm}^2</math></p>
2)	<p>A circular pond has a diameter of <math>5.36\text{ m}</math> to the nearest centimetre.</p> <p>a) Find the upper bound of the circumference of the pond. Give your answer to 3 decimal places.</p> <p>b) Find the lower bound of the area of the pond. Give your answer to 3 decimal places.</p>	<p>a) <math>16.855\text{ cm}</math></p> <p>b) <math>22.522\text{ cm}^2</math></p>
3)	<p>The diagram shows the right-angled triangle <math>ABC</math> where <math>AB = 12\text{ cm}</math> and <math>AC = 9\text{ cm}</math>. Both measurements have been rounded to the nearest centimetre.</p>  <p>a) Find the lower bound for the length <math>BC</math>.</p> <p>b) Find the error interval for the angle <math>BAC</math>.</p>	<p>a) <math>6.481\text{ cm}</math></p> <p>b) <math>34.301^\circ \leq BAC &lt; 47.156^\circ</math></p>



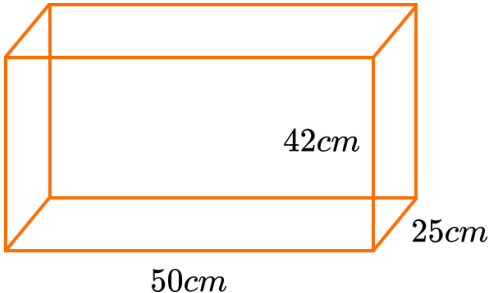
## Upper and Lower Bounds - Answers

<b>4)</b>	<p>Using the formula <math>P = \frac{F}{A}</math></p> <p><b>a)</b> Find the upper bound of <math>P</math> if <math>F = 45.72</math> to 2 d.p and <math>A = 5.8</math> to 1 d.p.</p> <p><b>b)</b> If <math>F = 5.21</math> to 2 d.p and <math>P = 1.67</math> to 2 d.p. By considering bounds, find the value of <math>A</math> to a suitable degree of accuracy.</p>	<p><b>a)</b> <math>45.725 \div 5.75 = 7.9521\dots</math></p> <p><b>b)</b> UB = 3.1321... LB = 3.1074... A = 3.1 to 1 d.p</p>
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## Upper and Lower Bounds - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	$E = mgh$ $m = 4$ correct to the nearest integer. $g = 9.81$ correct to 3 significant figures. $h = 2.4$ correct to 1 decimal place.		
(a)	Find the lower bound for $E$ .	(a) 3.5 or 9.805 or 2.35	(1)
		80.646125	(1)
(b)	Find the upper bound for $E$ .	(b) 4.5 or 9.815 or 2.45	(1)
		108.210375	(1)
2)	A rectangular field has a length of 260 metres to 2 significant figures and a width of 145 metres to the nearest metre.		
(a)	Find the lower bound of the area of the field.	(a) 255 or 144.5	(1)
		$36847.5 \text{ m}^2$	(1)
(b)	Find the upper bound of the perimeter of the field.	(b) 265 or 145.5	(1)
		$821 \text{ m}$	(1)

## Upper and Lower Bounds - Mark Scheme

<b>3)</b>	<p>A container in the shape of a cuboid is to be filled with soil. The cuboid is 50 <i>cm</i> by 42 <i>cm</i> by 25 <i>cm</i>, where the measurements are correct to the nearest <i>cm</i>.</p> <p>The container will be filled using a jug that can hold 2500 <i>ml</i> to the nearest 100 <i>ml</i>.</p>  <p>What is the minimum number of full jugs that may be required to fill the container?</p>	<p>49.5 or 41.5 or 24.5</p> <p>LB (volume)  <math>49.5 \times 41.5 \times 24.5</math>  <math>(= 50329.125)</math></p> <p>2550</p> <p><math>\frac{49.5 \times 41.5 \times 24.5}{2550} (= 19.7369...)</math></p> <p>20 jugs</p>	<p><b>(1)</b></p> <p><b>(1)</b></p> <p><b>(1)</b></p> <p><b>(1)</b></p> <p><b>(1)</b></p>
<b>4)</b>	$K = \sqrt{\frac{M}{H}}$ <p><math>M = 8.42</math> to 3 significant figures  <math>H = 2.64</math> to 3 significant figures.</p> <p>By considering bounds, work out the value of <math>K</math> to a suitable degree of accuracy, justifying your answer.</p>	<p>8.425 or 8.415 or 2.645 or 2.635 seen</p> <p><math>\sqrt{\frac{8.425}{2.635}}</math> or <math>\sqrt{\frac{8.415}{2.645}}</math></p> <p>1.788111... or 1.78366</p> <p><math>K = 1.8</math></p> <p>Both LB and UB round to 1.8 to 2 sf</p>	<p><b>(1)</b></p> <p><b>(1)</b></p> <p><b>(1)</b></p> <p><b>(1)</b></p> <p><b>(1)</b></p>

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