THIRD SPACE LEARNING

# Mathematical Proof - Worksheet

**Skill**

**Group A - Algebraic proof**

Prove algebraically that:

**1)** $3(4x + y) + 2(2x - 5y) \equiv 16x - 7y$

**2)** $(n + 4)^2 \equiv n^2 + 8n + 16$

**3)** $(n - 2)^2 - (n + 10) \equiv (n - 6)(n + 1)$

**4)** The sum of an odd number and an even number is odd.

**5)** The product of any two even numbers is even.

**6)** The sum of four consecutive whole numbers is always even.

**7)** $(n + 1)^2 - (n - 1)^2 - 1$ is odd for all positive integer values of $n$.

**8)** $(2n - 3)(5n - 4) - (4n - 3)(n - 4)$ is always even.

**9)** $2n(3n + 4) + (n - 4)^2$ is positive for all values of $n$.

**10)** $35n$ is always a multiple of 7.

**11)** $24n + 32$ is always a multiple of 8.

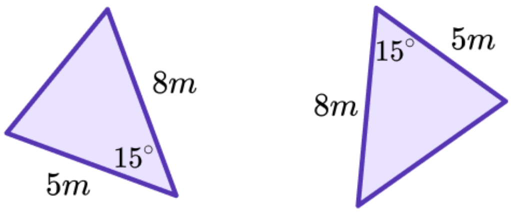
**12)** $(5n + 1)^2 + 2(4 - 10n) + 1$ is always a multiple of 5.

# Mathematical Proof - Worksheet

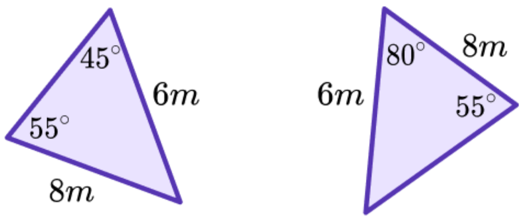
**Group B - Geometric proof (congruent triangles)**

Are these pairs of triangles congruent? If they are, state the condition that proves congruence:

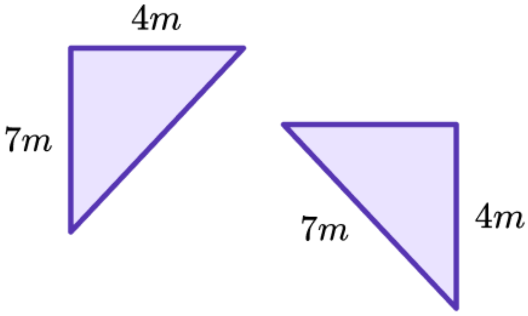
**1)**



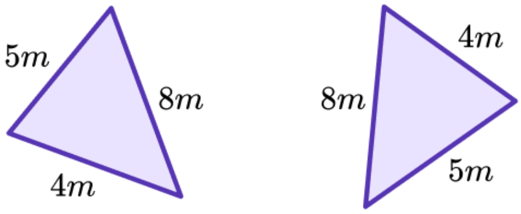

**2)**



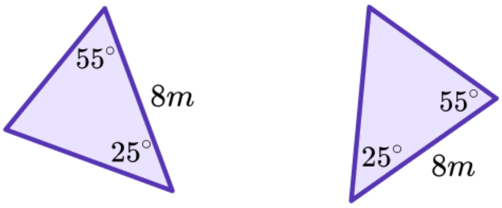

**3)**



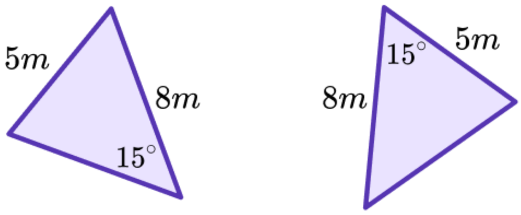

**4)**



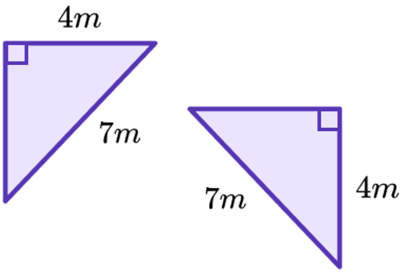

**5)**



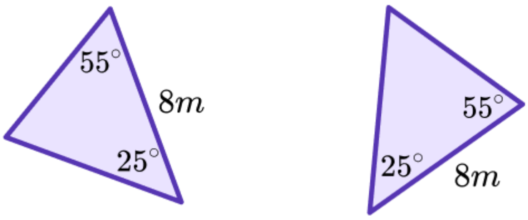

**6)**



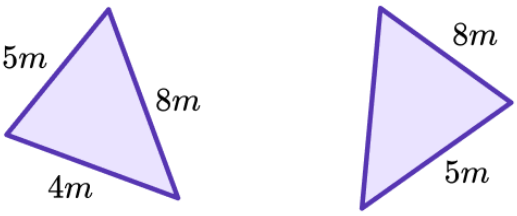

**7)**



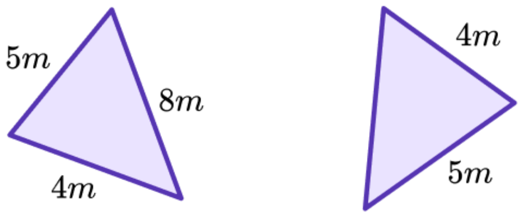

**8)**



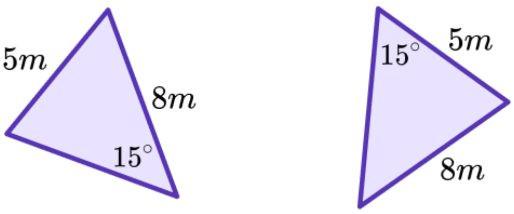

**9)**



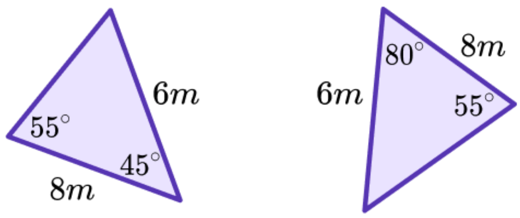

**10)**




**11)**




**12)**

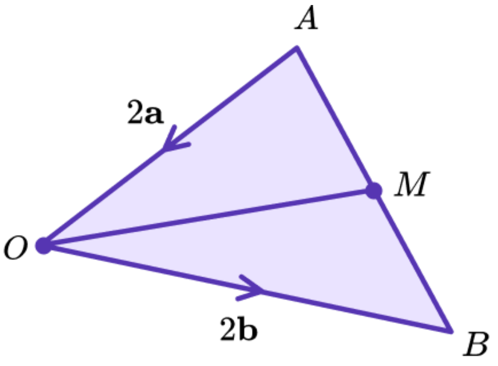


# Mathematical Proof - Worksheet

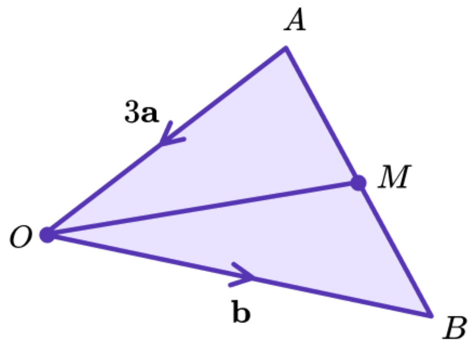
**Group C - Geometric proof (vectors)**

Prove the following:

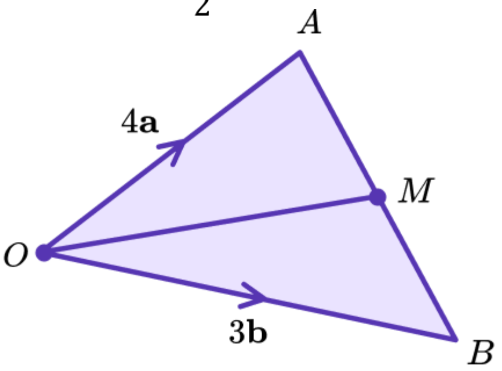
**1)** $M$ is the midpoint of the line.
Prove $\overrightarrow{AM} = \boldsymbol{a} + \boldsymbol{b}$ .



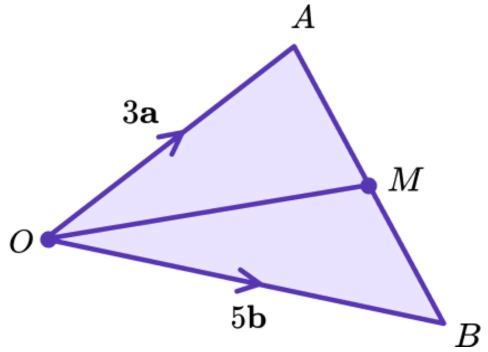

**2)** $M$ is the midpoint of the line.
Prove $\overrightarrow{AM} = \frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$ .



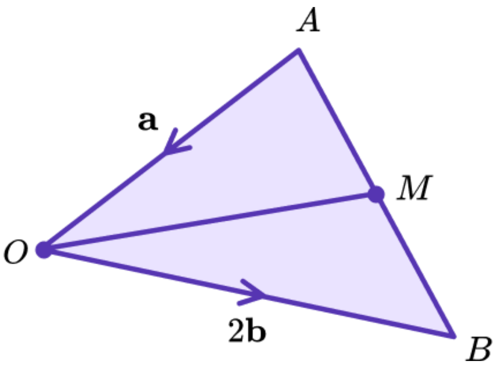

**3)** $M$ is the midpoint of the line.
Prove $\overrightarrow{AM} = \frac{3}{2}\boldsymbol{b} - 2\boldsymbol{a}$.



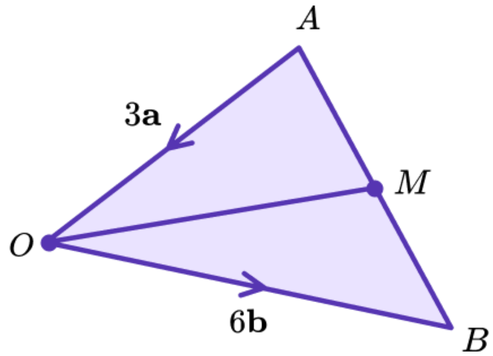

**4)** $AM : MB = 1 : 1$.
Prove $\overrightarrow{AM} = 2.5\boldsymbol{b} - 1.5\boldsymbol{a}$




**5)** $AM : MB = 1 : 2$.
Prove $\overrightarrow{AM} = \frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$.

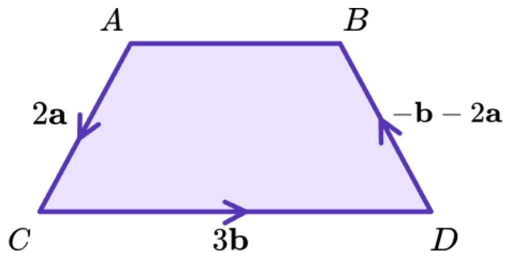



**6)** $AM : MB = 1 : 2$.
Prove $\overrightarrow{AM} = \boldsymbol{a} + 2\boldsymbol{b}$.

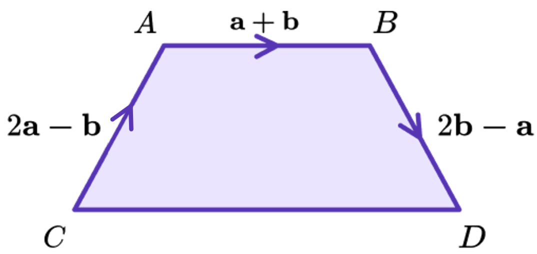


# Mathematical Proof - Worksheet

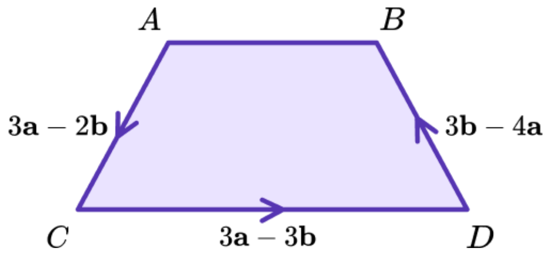
**7)** Prove $AB$ is parallel to $CD$.



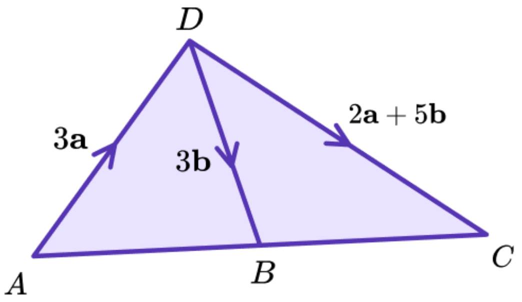

**8)** Prove $CD$ is parallel to $AB$.



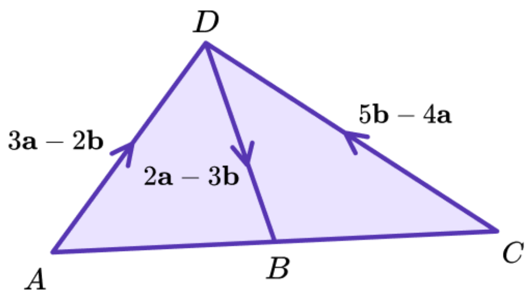

**9)** Prove $AB$ is parallel to $CD$.



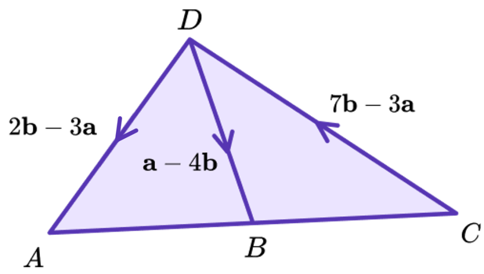

**10)** Prove $ABC$ is a straight line.




**11)** Prove $ABC$ is a straight line.




**12)** Prove $ABC$ is a straight line.



# Mathematical Proof - Worksheet

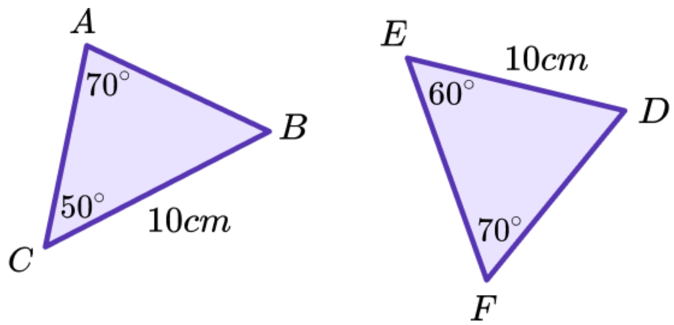
## Applied

**1)** Show that the sum of any five consecutive numbers is a multiple of 5.

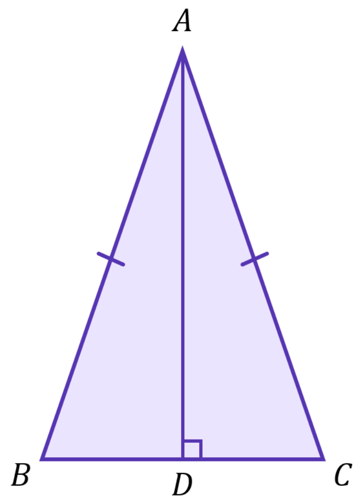
**2)** **(a)** Expand and simplify $(a - b)(a + b - 1)$.

**(b)** Show that $(2a - 1)^2 - (2b - 1)^2 = 4(a - b)(a + b - 1)$.

**3)** Prove that the triangles $ABC$ and $DEF$ are congruent.
Diagrams not drawn to scale.




**4)** The triangle $ABC$ is an isosceles triangle.
$AD$ is perpendicular to $BC$.

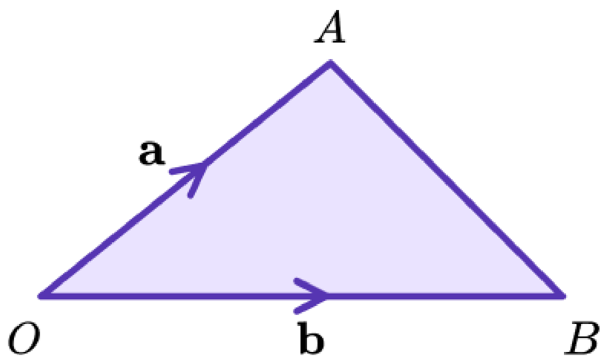



**(a)** Use congruent triangles to prove that $DB = DC$.

**(b)** $AB = 10cm$
$BC = 12cm$
Work out the area of the triangle $ABC$.

# Mathematical Proof - Worksheet

**5)**




$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = \boldsymbol{b}$

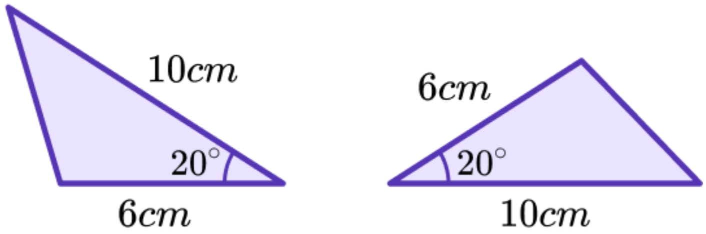
$M$ is the point on $AB$ such that $AM: MB = 1: 3$

$\overrightarrow{OM} = k(3\boldsymbol{a} + \boldsymbol{b})$

Find the value of $k$.

# Mathematical Proof - Exam Questions

**1)**




Circle the reason why these triangles are congruent.

RHS ASA SSS SAS

**(1 mark)**

---

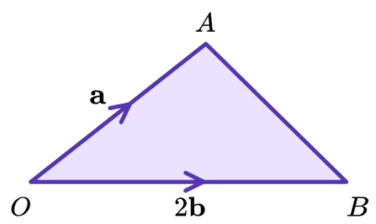
**2)** Prove that $(5n + 3)^2 - (5n - 3)^2$ is a multiple of 12, for all positive integer values of $n$.

..................

**(3 marks)**

---

**3)**




$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = 2\boldsymbol{b}$

$Q$ is the point on $AB$ such that $AQ: QB = 3: 2$

$\overrightarrow{OQ} = k(\boldsymbol{a} + 3\boldsymbol{b})$

Find the value of $k$

..................

**(4 marks)**

# Mathematical Proof - Exam Questions

**4)** Prove algebraically that the difference between the squares of any two consecutive odd numbers is equal to twice the sum of these two integers.

....................

**(5 marks)**

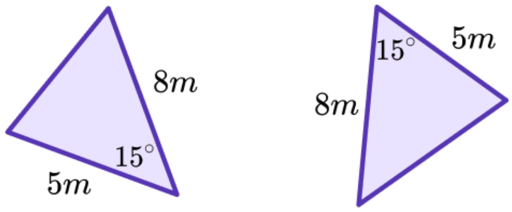
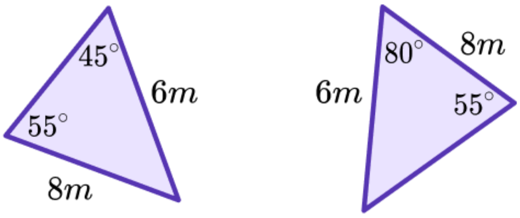
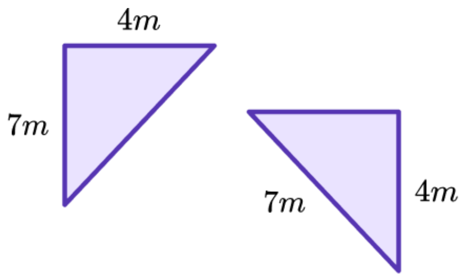
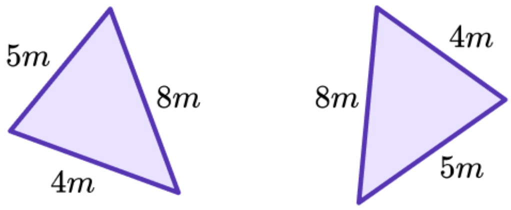
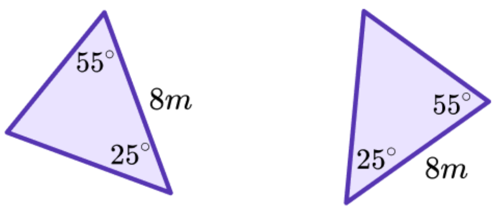
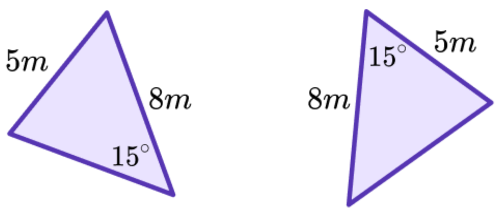
# Mathematical Proof - Answers

| | Question | Answer |
|---|---|---|
| | Skill Questions | |
| Group A | Prove algebraically that:<br><br>**1)** $3(4x + y) + 2(2x - 5y) \equiv 16x - 7y$ | **1)** $3(4x + y) + 2(2x - 5y)$<br>$= 12x + 3y + 4x - 10y$<br>$= 16x - 7y$ |
| | **2)** $(n + 4)^2 \equiv n^2 + 8n + 16$ | **2)** $(n + 4)^2$<br>$= n^2 + 4n + 4n + 16$<br>$= n^2 + 8n + 16$ |
| | **3)** $(n - 2)^2 - (n + 10) \equiv (n - 6)(n + 1)$ | **3)** $(n - 2)^2 - (n + 10)$<br>$= n^2 - 4n + 4 - n - 10$<br>$= n^2 - 5n - 6$<br>$= (n - 6)(n + 1)$ |
| | **4)** The sum of an odd number and an even number is odd. | **4)** $2n + (2n + 1)$<br>$= 4n + 1$<br>$= 2(2n) + 1$<br>Even + 1<br>$\therefore$ Odd |
| | **5)** The product of any two even numbers is even. | **5)** $2n \times 2n$<br>$= 4n^2$<br>$= 2(2n^2)$<br>$\therefore$ Even |
| | **6)** The sum of four consecutive whole numbers is always even. | **6)** $n + (n + 1) + (n + 2) + (n + 3)$<br>$= 4n + 6$<br>$= 2(2n + 3)$<br>$\therefore$ Even |

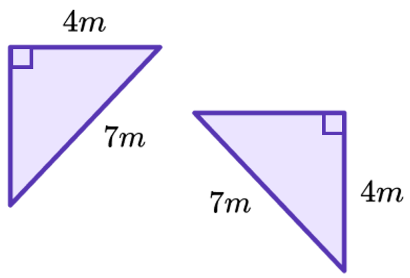
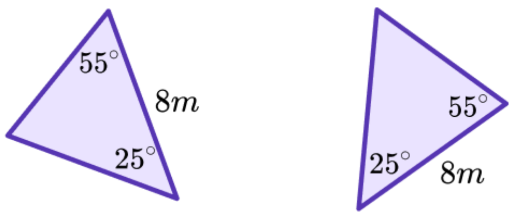
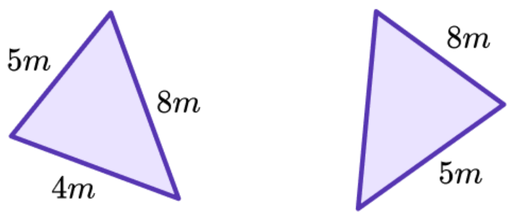
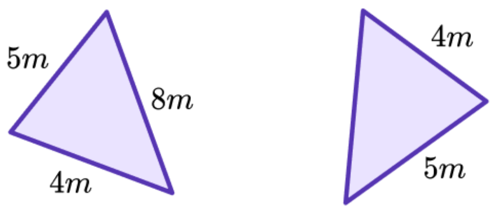
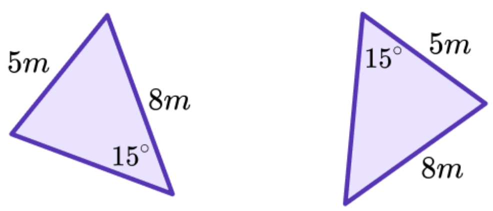
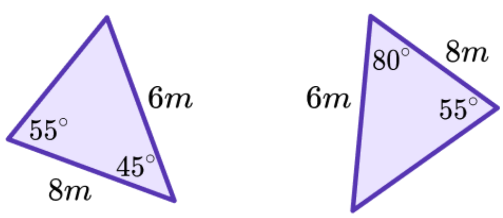
## Mathematical Proof - Answers

| | | |
|---|---|---|
| Group A contd | **7)** $(n+1)^2-(n-1)^2-1$ is odd for all positive integer values of $n$. | **7)** $(n+1)^2-(n-1)^2-1$<br>$=n^2+2n+1-$<br>$(n^2-2n+1)-1$<br>$=n^2+2n+1-$<br>$n^2+2n+1-1$<br>$=4n-1$<br>$=2(2n)-1$<br>Even -1<br>∴ Odd |
| | **8)** $(2n-3)(5n-4)-$ $(4n-3)(n-4)$ is always even. | **8)** $(2n-3)(5n-4)-$<br>$(4n-3)(n-4)$<br>$=10n^2-8n-15n+12-$<br>$(4n^2-16n-3n+12)$<br>$=10n^2-8n-15n+12-$<br>$4n^2+16n+3n-12$<br>$=6n^2-4n$<br>$=2(3n^2-2n)$<br>∴ Even |
| | **9)** $2n(3n+4)+(n-4)^2$ is positive for all values of $n$. | **9)** $2n(3n+4)+(n-4)^2$<br>$=6n^2+8n+n^2-8n+16$<br>$=7n^2+16$<br>$n^2\geq 0$ always<br>∴ $7n^2\geq 0$<br>∴ $7n^2+16>0$ |
| | **10)** $35n$ is always a multiple of 7. | **10)** $35n=7\times 5n$<br>∴ multiple of 7 |
| | **11)** $24n+32$ is always a multiple of 8. | **11)** $24n+32=8(3n+4)$<br>∴ multiple of 8 |
| | **12)** $(5n+1)^2+2(4-10n)+1$ is always a multiple of 5. | **12)** $(5n+1)^2+2(4-10n)+1$<br>$25n^2+10n+1+8-20n+1$<br>$=25n^2-10n+10$<br>$=5(5n^2-2n+5)$ |

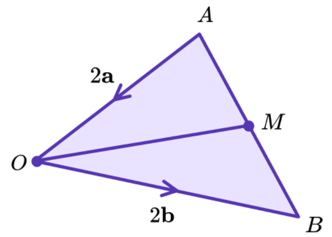
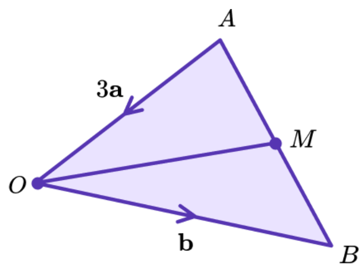
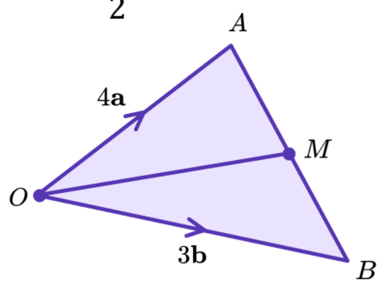
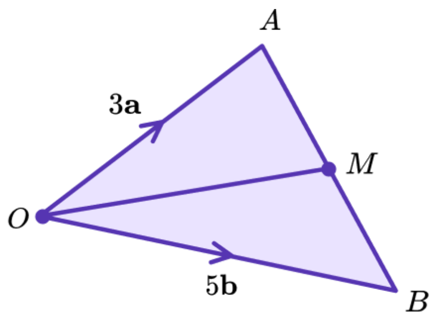
# Mathematical Proof - Answers

| | | |
|---|---|---|
| Group B | Are these pairs of triangles congruent? If they are, state the condition that proves congruence: | |
| | **1)**   | **1)** Yes, SAS |
| | **2)**   | **2)** Yes, SAS (or ASA) |
| | **3)**   | **3)** No |
| | **4)**   | **4)** Yes, SSS |
| | **5)**   | **5)** Yes, ASA |
| | **6)**   | **6)** No |

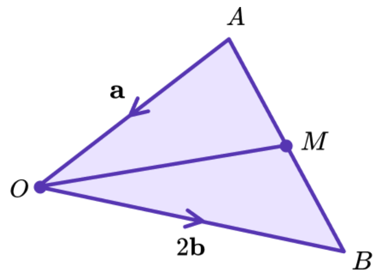
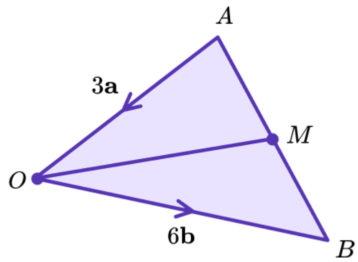
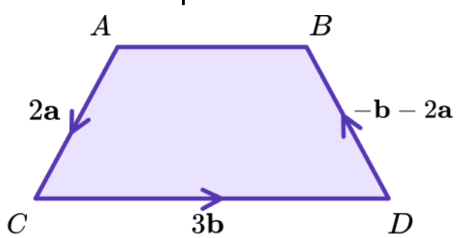
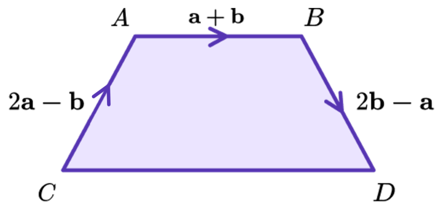
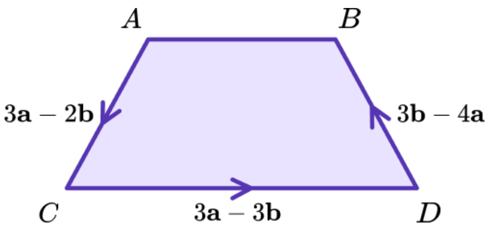
# Mathematical Proof - Answers

| | | |
|---|---|---|
| Group B contd | **7)**   | **7)** Yes, RHS |
| | **8)**   | **8)** Yes, ASA |
| | **9)**   | **9)** No |
| | **10)**   | **10)** No |
| | **11)**   | **11)** No |
| | **12)**   | **12)** Yes, ASA (or SAS) |

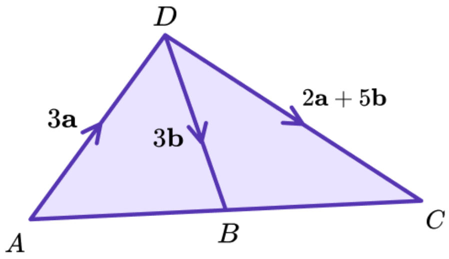
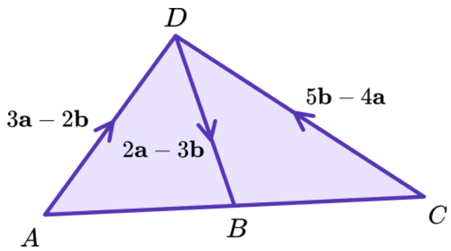
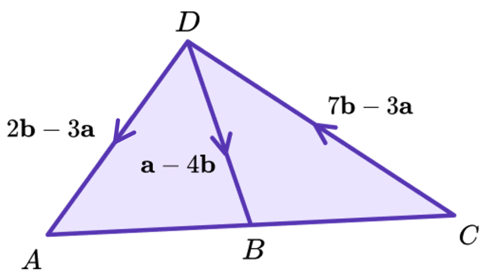
# Mathematical Proof - Answers

| Group C | Prove the following: | |
|---|---|---|
| | **1)** $M$ is the midpoint of the line.<br>Prove $\overrightarrow{AM} = \boldsymbol{a} + \boldsymbol{b}$ .<br> | **1)** $\frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(2\boldsymbol{a} + 2\boldsymbol{b}) = \boldsymbol{a} + \boldsymbol{b}$ |
| | **2)** $M$ is the midpoint of the line.<br>Prove $\overrightarrow{AM} = \frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$.<br> | **2)** $\frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(3\boldsymbol{a} + \boldsymbol{b}) = \frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$ |
| | **3)** $M$ is the midpoint of the line.<br>Prove $\overrightarrow{AM} = \frac{3}{2}\boldsymbol{b} - 2\boldsymbol{a}$ .<br> | **3)** $\frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(-4\boldsymbol{a} + 3\boldsymbol{b}) = -2\boldsymbol{a} + \frac{3}{2}\boldsymbol{b}$ |
| | **4)** $AM : MB = 1 : 1$.<br>Prove $\overrightarrow{AM} = 2.5\boldsymbol{b} - 1.5\boldsymbol{a}$.<br> | **4)** $\frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(-3\boldsymbol{a} + 5\boldsymbol{b}) = -\frac{3}{2}\boldsymbol{a} + \frac{5}{2}\boldsymbol{b}$ |

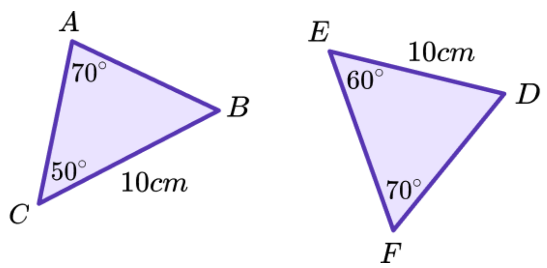
# Mathematical Proof - Answers

| | | |
|---|---|---|
| Group C contd | **5)** $AM : MB = 1 : 2$.<br>Prove $\overrightarrow{AM} = \frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$.<br> | **5)** $\frac{1}{3}\overrightarrow{AB} = \frac{1}{3}(\boldsymbol{a} + 2\boldsymbol{b}) = \frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$ |
| | **6)** $AM : MB = 1 : 2$.<br>Prove $\overrightarrow{AM} = \boldsymbol{a} + 2\boldsymbol{b}$.<br> | **6)** $\frac{1}{3}\overrightarrow{AB} = \frac{1}{3}(3\boldsymbol{a} + 6\boldsymbol{b}) = \boldsymbol{a} + 2\boldsymbol{b}$ |
| | **7)** Prove $AB$ is parallel to $CD$.<br> | **7)** $\overrightarrow{AB} = 2\boldsymbol{a} + 3\boldsymbol{b} - \boldsymbol{b} - 2\boldsymbol{a}$<br>$= 2\boldsymbol{b}$ |
| | **8)** Prove $CD$ is parallel to $AB$.<br> | **8)** $\overrightarrow{CD} = 2\boldsymbol{a} - \boldsymbol{b} + \boldsymbol{a} + \boldsymbol{b} + 2\boldsymbol{b} - \boldsymbol{a}$<br>$= 2\boldsymbol{a} + 2\boldsymbol{b}$<br>$= 2(\boldsymbol{a} + \boldsymbol{b})$ |
| | **9)** Prove $AB$ is parallel to $CD$.<br> | **9)** $\overrightarrow{AB} = 3\boldsymbol{a} - 2\boldsymbol{b} + 3\boldsymbol{a} - 3\boldsymbol{b} + 3\boldsymbol{b} - 4\boldsymbol{a}$<br>$= 2\boldsymbol{a} - 2\boldsymbol{b}$<br>$= 2(\boldsymbol{a} - \boldsymbol{b})$<br>$\overrightarrow{CD} = 3(\boldsymbol{a} - \boldsymbol{b})$ |

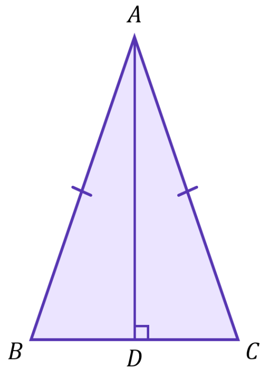
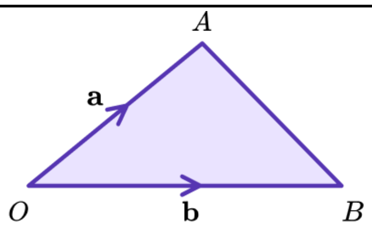
# Mathematical Proof - Answers

| | | |
|---|---|---|
| | **10)** Prove $ABC$ is a straight line.<br><br> | **10)** $\overrightarrow{AB} = 3\boldsymbol{a} + 3\boldsymbol{b} = 3(\boldsymbol{a} + \boldsymbol{b})$<br>$\overrightarrow{AC} = 3\boldsymbol{a} + 2\boldsymbol{a} + 5\boldsymbol{b} = 5(\boldsymbol{a} + \boldsymbol{b})$<br><br>Both vectors move in the same direction (but have different lengths). They both start from the same point $A$, so $ABC$ is a straight line. |
| | **11)** Prove $ABC$ is a straight line<br><br> | **11)**<br>$\overrightarrow{AB} = 3\boldsymbol{a} - 2\boldsymbol{b} + 2\boldsymbol{a} - 3\boldsymbol{b} = 5(\boldsymbol{a} - \boldsymbol{b})$<br>$\overrightarrow{AC} = 3\boldsymbol{a} - 2\boldsymbol{b} - 5\boldsymbol{b} + 4\boldsymbol{a} = 7(\boldsymbol{a} - \boldsymbol{b})$<br><br>Both vectors move in the same direction (but have different lengths). They both start from the same point $A$, so $ABC$ is a straight line. |
| | **12)** Prove $ABC$ is a straight line<br><br> | **12)**<br>$\overrightarrow{AB} = -2\boldsymbol{b} + 3\boldsymbol{a} + \boldsymbol{a} - 4\boldsymbol{b} = 2(2\boldsymbol{a} - 3\boldsymbol{b})$<br>$\overrightarrow{AC} = -2\boldsymbol{b} + 3\boldsymbol{a} - 7\boldsymbol{b} + 3\boldsymbol{a} = 3(2\boldsymbol{a} - 3\boldsymbol{b})$<br><br>Both vectors move in the same direction (but have different lengths). They both start from the same point $A$, so $ABC$ is a straight line. |

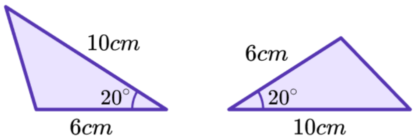
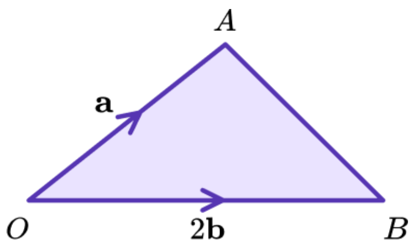
# Mathematical Proof - Answers

| | Question | Answer |
|---|---|---|
| | Applied Questions | |
| **1)** | Show that the sum of any five consecutive numbers is a multiple of 5. | $n + (n + 1) + (n + 2) + (n + 3) + (n + 4)$<br>$= 5n + 10$<br>$= 5(n + 2)$<br>∴ Multiple of 5 |
| **2)** | **a)** Expand and simplify $(a + b)(a - b + 1)$<br>**b)** Show that $3a(a + 1) + 3b(1 - b) \equiv 3(a + b)(a - b + 1)$ | **a)** $a^2 + a - b^2 + b$<br>**b)** $3a(a + 1) + 3b(1 - b)$<br>$= 3a^2 + 3a + 3b - 3b^2$<br>$= 3(a^2 + a + b - b^2)$<br>$= 3(a^2 + a - b^2 + b)$<br>$= 3(a + b)(a - b + 1)$ |
| **3)** | Prove that the triangles $ABC$ and $DEF$ are congruent.<br>Diagrams not drawn to scale<br> | Angle ABC = 180 - (70 + 50) = 60°<br>Angle EDF = 180 - (70 + 60) = 50°<br><br>Hence ABC is congruent to DEF by ASA |

# Mathematical Proof - Answers

| | | |
|---|---|---|
| **4)** | The triangle $ABC$ is an isosceles triangle. $AD$ is perpendicular to $BC$.<br><br> | |
| | **a)** Use congruent triangles to prove that $DB = DC$. | **a)** Consider triangles ADB and ADC.<br>AD=AD since common side<br>AB=AC since isosceles triangle<br>Angle ADB = 90° = Angle ADC<br>Congruence by RHS<br>Hence DB = DC |
| | **b)** $AB = 10cm$<br>$BC = 12cm$<br>Work out the area of the triangle $ABC$. | **b)** $48cm^2$ |
| **5)** | <br><br>$\overrightarrow{OA} = \boldsymbol{a}$<br>$\overrightarrow{OB} = \boldsymbol{b}$<br>$M$ is the point on $AB$ such that<br>$AM: MB = 1: 3$<br>$\overrightarrow{OM} = k(3\boldsymbol{a} + \boldsymbol{b})$<br>Find the value of $k$. | $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$<br>$\overrightarrow{AM} = \frac{1}{4}(\mathbf{b} - \mathbf{a})$<br>$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$<br>$= \mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a})$<br>$= \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$<br>$= \frac{1}{4}(3\mathbf{a} + \mathbf{b})$<br>$K = \frac{1}{4}$ |

# Mathematical Proof - Mark Scheme

| | Question | Answer | |
|---|---|---|---|
| | Exam Questions | | |
| **1)** | <br>Circle the reason why these triangles are congruent.<br>RHS ASA SSS SAS | SAS | **(1)** |
| **2)** | Prove that $(5n + 3)^2 - (5n - 3)^2$ is a multiple of 12, for all positive integer values of $n$. | $(5n + 3)^2 - (5n - 3)^2$<br>$= 25n^2 + 30n + 9 - (25n^2 - 30n + 9)$<br>$= 25n^2 + 30n + 9 - 25n^2 + 30n - 9$<br>$= 60n$<br>$= 12 \times 5n$ **and** multiple of 12 | <br>**(1)**<br><br>**(1)**<br>**(1)** |
| **3)** | <br>$\overrightarrow{OA} = \boldsymbol{a}$<br>$\overrightarrow{OB} = 2\boldsymbol{b}$<br>$Q$ is the point on $AB$ such that $AQ : QB = 3 : 2$<br>$\overrightarrow{OQ} = k(\boldsymbol{a} + 3\boldsymbol{b})$<br>Find the value of $k$. | $\overrightarrow{AB} = -\mathbf{a} + 2\mathbf{b}$<br>$\overrightarrow{AQ} = \frac{3}{5}(-\mathbf{a} + 2\mathbf{b})$<br>$\overrightarrow{OQ} = \mathbf{a} + \frac{3}{5}(-\mathbf{a} + 2\mathbf{b})$<br>$= \frac{2}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}$<br>$= \frac{2}{5}(\mathbf{a} + 3\mathbf{b})$<br>$K = \frac{2}{5}$ | **(1)**<br>**(1)**<br>**(1)**<br><br><br>**(1)** |

THIRD SPACE LEARNING

# Mathematical Proof - Mark Scheme

| | | | |
|---|---|---|---|
| **4)** | Prove algebraically that the difference between the squares of any two consecutive odd numbers is equal to twice the sum of these two integers. | $(2n + 1)^2 - (2n - 1)^2$<br>$= 4n^2 + 4n + 1 - (4n^2 - 4n + 1)$<br>$= 4n^2 + 4n + 1 - 4n^2 + 4n - 1$<br>$= 8n$<br><br>$2n + 1 + 2n - 1 = 4n$<br>$4n \times 2 = 8n$ | **(1)**<br><br>**(1)**<br>**(1)**<br><br>**(1)**<br>**(1)** |