Algebraic Fractions - Worksheet

Skill

Group A - Solve equations: single algebraic fraction (linear)

Calculate the value for x for each linear equation.

1) $\frac{2x}{3} = 4$

2) $\frac{x}{5} + 3 = 5$

3) $\frac{x}{4} + 7 = 9$

4) $\frac{15}{x} - 7 = -12$

5) $\frac{2+5x}{7} = 6$

6) $\frac{4-7x}{11} = -6$

7) $\frac{5-4x}{15} = -5$

8) $5 - \frac{x}{3} = 8$

9) $3 - \frac{4x}{5} = 11$

10) $26 - \frac{2x}{3} = 14$

11) $7 + \frac{8}{x} = -9$

12) $-\left(\frac{5x-20}{8}\right) = 10$

Group B - Solve equations: algebraic fractions (quadratic)

Calculate the value(s) for x for each quadratic equation.

1) $x + \frac{5}{x} = 6$

2) $x + \frac{6}{x} = 5$

3) $x + \frac{12}{x} = 7$

4) $2x + \frac{5}{x} = 7$

5) $7x - \frac{6}{x+1} = -26$

6) $5x - \frac{4}{x-2} = -9$

7) $x - 1 = 7 - \frac{15}{2x+1}$

8) $2x = \frac{x-3}{x+1} - 12$

9) $\frac{12}{6-x} + x = 2$

10) $\frac{x}{0.5} + \frac{1.5}{x} = 4$

11) $\frac{1}{2}\left(5x - \frac{7}{x+4}\right) = 7$

12) $\frac{x}{3}\left(\frac{x+7}{-5x}\right) = -\frac{1}{3}$

Algebraic Fractions - Worksheet

Group C - Solve equations: simplifying quadratics

Simplify the algebraic fractions to calculate the value(s) for x.

1) $\frac{5x+10}{2x} = 3$

2) $\frac{x^2+4x}{3x+12} = 12$

3) $\frac{x^2-4x}{x^2} = 3$

4) $x + \frac{x}{x-2} = 6$

5) $\frac{5x+10}{x^2+5x+6} = 1$

6) $\frac{2x+2}{x^2+4x+3} = \frac{1}{2}$

7) $\frac{7x+28}{x^2+x-12} = 3.5$

8) $\frac{3x-3}{x^2-1} = -1.5$

9) $\frac{x^2-4}{2x^2-3x-2} = 1$

10) $\frac{4x^2+12x+9}{10x+15} = 5$

11) $\frac{x}{x+1} + \frac{x}{x+4} = 1$

12) $\frac{3x^2+13x+4}{x^2+8x+16} = 20$

Algebraic Fractions - Worksheet

Applied

1) **(a)** Solve $\frac{x-4}{2}=\frac{6+x}{7}$.

(b) Solve $\frac{x+3}{5}=\frac{6-x}{40}$.

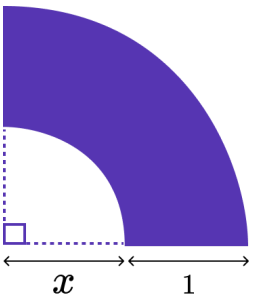
2) **(a)** Find the solution to the equation $\frac{x^2-x-30}{x-6}= 10$.

(b) Show that the equation $\frac{x^2+4x-21}{x-3}= 10$ has no solutions.

3) **(a)** Show that $x =- 1$ is a solution to $\frac{5}{x}+\frac{6}{x-1}+\frac{1}{x^2}=- 7$.

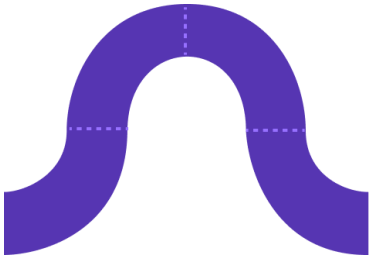
(b) Rearrange the equation into the form $7x^3 + ax^2 + bx + c = 0$, hence state the integer values of a, b, and c.

4) **(a)** The shape below has an area of $2\pi xcm^2$. Show that $x =\frac{1}{6}$.



Not drawn to scale

(b) A logo is created using multiple copies of the same image in part (a).



Calculate the area of the logo design, correct to 2 decimal places.

Algebraic Fractions - Exam Questions

1) **(a)** Simplify the algebraic fraction. ..... **(3)**

$$\frac{6x^2y^2}{8x^3y} \div 3xy$$

(b) Hence solve the equation. ..... **(4)**

(7 marks)

$$\frac{6x^2y^2}{8x^3y} \div 3xy = 2x$$

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2) Simplify fully. Write your answer in the form $\frac{a+b\sqrt{3}}{c}$ where a,b, and c are integers. $\frac{3+\sqrt{3}}{6+\sqrt{3}} + \sqrt{3}$. ..... **(5 marks)**

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3) **(a)** Simplify the fraction. ..... **(3)**

$$\frac{12x^2+30x-18}{24x^2-6}$$

(b) State the value of x for which the fraction is undefined. ..... **(1)**

(4 marks)

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4) **(a)** Simplify fully. ..... **(2)**

$$\frac{3}{x^2} + \frac{2}{3x^2} + \frac{3}{5x^2}$$

(b) Hence solve $\frac{3}{x^2} + \frac{2}{3x^2} + \frac{3}{5x^2} = \frac{x}{15}$. ..... **(3)**

(5 marks)

Algebraic Fractions - Answers

	Question	Answer
	Skill Questions	
Group A	Calculate the value for x for each linear equation. **1)** $\frac{2x}{3} = 4$ **2)** $\frac{x}{5} + 3 = 5$ **3)** $\frac{x}{4} + 7 = 9$ **4)** $\frac{15}{x} - 7 = -12$ **5)** $\frac{2+5x}{7} = 6$ **6)** $\frac{4-7x}{11} = -6$ **7)** $\frac{5-4x}{15} = -5$ **8)** $5 - \frac{x}{3} = 8$ **9)** $3 - \frac{4x}{5} = 11$ **10)** $26 - \frac{2x}{3} = 14$ **11)** $7 + \frac{8}{x} = -9$ **12)** $-\left(\frac{5x-20}{8}\right) = 10$	**1)** $x = 6$ **2)** $x = 10$ **3)** $x = 8$ **4)** $x = -3$ **5)** $x = 8$ **6)** $x = 10$ **7)** $x = 20$ **8)** $x = -9$ **9)** $x = -10$ **10)** $x = 18$ **11)** $x = -0.5$ **12)** $x = -12$
Group B	Calculate the value for x for each quadratic equation. **1)** $x + \frac{5}{x} = 6$ **2)** $x + \frac{6}{x} = 5$ **3)** $x + \frac{12}{x} = 7$	**1)** $x = 5$ or $x = 1$ **2)** $x = 2$ or $x = 3$ **3)** $x = 3$ or $x = 4$

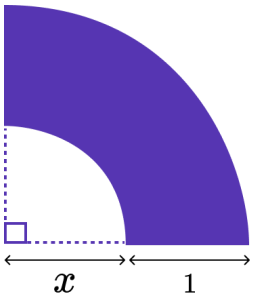
Algebraic Fractions - Answers

	4) $2x + \frac{5}{x} = 7$ **5)** $7x - \frac{6}{x+1} = -26$ **6)** $5x - \frac{4}{x-2} = -9$ **7)** $x - 1 = 7 - \frac{15}{2x+1}$ **8)** $2x = \frac{x-3}{x+1} - 12$ **9)** $\frac{12}{6-x} + x = 2$ **10)** $\frac{x}{0.5} + \frac{1.5}{x} = 4$ **11)** $\frac{1}{2}(5x - \frac{7}{x+4}) = 7$ **12)** $\frac{x^2}{3}(\frac{x+7}{-5x}) = -\frac{1}{3}$	**4)** $x = 1$ or $x = 2.5$ **5)** $x = -4$ or $x = -\frac{5}{7}$ **6)** $x = -2$ or $x = \frac{11}{5}$ **7)** $x = 7$ or $x = \frac{1}{2}$ **8)** $x = -5$ or $x = -\frac{3}{2}$ **9)** $x = 0$ or $x = 8$ **10)** $x = 0.5$ or $x = 1.5$ **11)** $x = 3$ or $x = -\frac{21}{5}$ **12)** $x = \frac{-7 \pm \sqrt{29}}{2}$
Group C	Simplify the algebraic fractions to calculate the value(s) for x. **1)** $\frac{5x+10}{2x} = 3$ **2)** $\frac{x^2+4x}{3x+12} = 12$ **3)** $\frac{x^2-4x}{x^2} = 3$ **4)** $x + \frac{x}{x-2} = 6$ **5)** $\frac{5x+10}{x^2+5x+6} = 1$ **6)** $\frac{2x+2}{x^2+4x+3} = \frac{1}{2}$ **7)** $\frac{7x+28}{x^2+x-12} = 3.5$ **8)** $\frac{3x-3}{x^2-1} = -1.5$	**1)** $x = 10$ **2)** $x = 36$ **3)** $x = -2$ **4)** $x = 4$ or $x = 3$ **5)** $x = 2$ **6)** $x = 1$ **7)** $x = 5$ **8)** $x = -3$

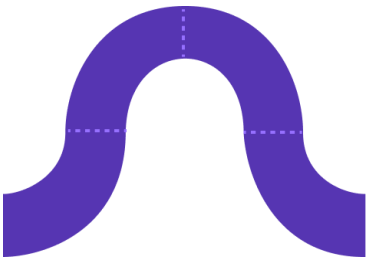
Algebraic Fractions - Answers

	9) $\frac{x^2-4}{2x^2-3x-2} = 1$ **10)** $\frac{4x^2+12x+9}{10x+15} = 5$ **11)** $\frac{x}{x+1} + \frac{x}{x+4} = 1$ **12)** $\frac{3x^2+13x+4}{x^2+8x+16} = -8$	**9)** $x = 1$ **10)** $x = 11$ **11)** $x = \pm 2$ **12)** $x = -3$

Algebraic Fractions - Answers

	Question	Answer
	Applied Questions	
1)	**(a)** Solve $\frac{x-4}{2}=\frac{6+x}{7}$. **(b)** Solve $\frac{x+3}{5}=\frac{6-x}{40}$.	**(a)** $x = 8$ **(b)** $x = -2$
2)	**(a)** Find the solution to the equation $\frac{x^2-x-30}{x-6}= 10$. **(b)** Show that the equation $\frac{x^2+4x-21}{x-3}= 10$ has no solutions.	**(a)** $x = 5$ (Cannot be $x = 6$ as the denominator would be 0). **(b)** $x = 3$ is the only solution but $x \neq 3$ as the denominator would equal 0.
3)	**(a)** Show that $x = -1$ is a solution to $\frac{5}{x}+\frac{6}{x-1}+\frac{1}{x^2}=-7$. **(b)** Rearrange the equation into the form $7x^3 + ax^2 + bx + c = 0$, hence state the integer values of a, b, and c.	**(a)** When $x = -1$ $\frac{5}{-1}+\frac{6}{-1-1}+\frac{1}{(-1)^2}$ $= -5 - 3 + 1 = -7$ **(b)** $7x^3 + 4x^2 - 4x - 1 = 0$ $a = 4,\ b = -4,\ c = -1$
4)	**(a)** The shape below has an area of $2\pi xcm^2$. Show that $x = \frac{1}{6}$.  Not drawn to scale	**(a)** $\frac{\pi(x+1)^2}{4}-\frac{\pi(x)^2}{4}= 2\pi x$ $(x + 1)^2 - x^2 = 8x$ $x^2 + 2x + 1 - x^2 = 8x$ $2x + 1 = 8x$ $6x = 1$ $x = \frac{1}{6}$

Algebraic Fractions - Answers

	(b)	A logo is created using multiple copies of the same image in part a).  Calculate the area of the logo design, correct to 2 decimal places.	**(b)**	$2\pi x \times 4 = 8\pi x$ As $x = \frac{1}{6}$ Area$= \frac{8\pi}{6}$ $= 4.19cm^2$

Algebraic Fractions - Mark Scheme

	Question	Answer	
	Exam Questions		
1) (a)	Simplify the algebraic fraction. $\frac{6x^2y^2}{8x^3y} \div 3xy$	**(a)** $\frac{6x^2y^2}{8x^3y} \div 3xy$ $= \frac{6x^2y^2}{8x^3y} \times \frac{1}{3xy} = \frac{6x^2y^2}{24x^4y^2}$ $= \frac{1}{4x^2}$	**(3)**
(b)	Hence solve the equation. $\frac{6x^2y^2}{8x^3y} \div 3xy = 2x$	**(b)** $\frac{1}{4x^2} = 2x$ $1 = 8x^3$ $x^3 = \frac{1}{8}$ $x = \frac{1}{2}$	**(4)**
2)	Simplify fully. $\frac{3+\sqrt{3}}{6+\sqrt{3}} + \sqrt{3}$ Write your answer in the form $\frac{a+b\sqrt{3}}{c}$ where a,b, and c are integers.	$\frac{3+\sqrt{3}}{6+\sqrt{3}} + \sqrt{3} = \frac{3+\sqrt{3}}{6+\sqrt{3}} + \frac{\sqrt{3}(6+\sqrt{3})}{(6+\sqrt{3})}$ $= \frac{3+\sqrt{3}+6\sqrt{3}+3}{6+\sqrt{3}}$ $= \frac{6+7\sqrt{3}}{6+\sqrt{3}}$ $= \frac{(6+7\sqrt{3})(6-\sqrt{3})}{(6+\sqrt{3})(6-\sqrt{3})}$ $= \frac{36+\sqrt{3}-21}{36-3}$ $= \frac{15+\sqrt{3}}{33}$ $a = 15,\ b = 1,\ c = 33$	**(5)**

Algebraic Fractions - Mark Scheme

3) (a)	Simplify the fraction. $\frac{12x^2+42x+18}{24x^2-6}$	**(a)** $\frac{12x^2+42x+18}{24x^2-6}=\frac{6(2x^2+7x+3)}{6(4x^2-1)}$ $=\frac{2x^2+7x+3}{4x^2-1}$ $=\frac{(2x+1)(x+3)}{(2x+1)(2x-1)}$ $=\frac{x+3}{2x-1}$	**(3)**
(b)	State the value of x for which the fraction is undefined.	**(b)** The fraction is undefined when the denominator is equal to 0. $2x-1=0$ $x=\frac{1}{2}$	**(1)**
4) (a)	Simplify fully $\frac{3}{x^2}+\frac{2}{3x^2}+\frac{3}{5x^2}$.	**(a)** $\frac{3}{x^2}+\frac{2}{3x^2}+\frac{3}{5x^2}=\frac{45}{5x^2}+\frac{10}{15x^2}+\frac{9}{15x^2}$ $=\frac{64}{15x^2}$	**(2)**
(b)	Hence solve $\frac{3}{x^2}+\frac{2}{3x^2}+\frac{3}{5x^2}=\frac{x}{15}$.	**(b)** $\frac{64}{15x^2}=\frac{x}{15}$ $64=\frac{15x^3}{15}$ $64=x^3$ $x=4$	**(3)**