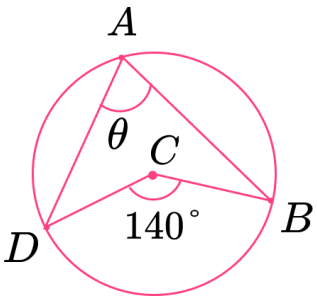
Circle Theorems - Worksheet

Skill

Group A - Angles at the Centre, Cyclic Quadrilaterals

Calculate the size of the missing angles marked θ

1)



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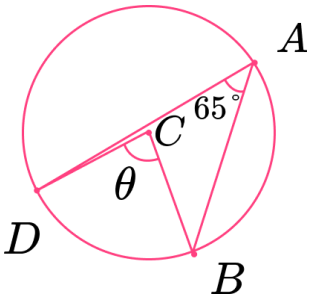
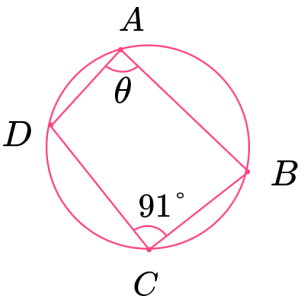


3)



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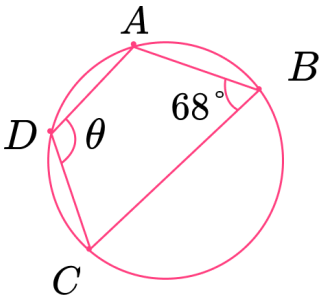

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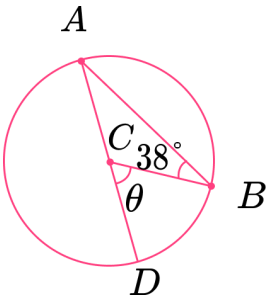

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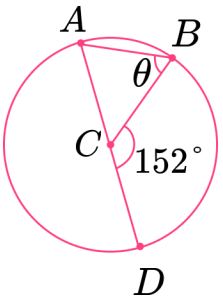
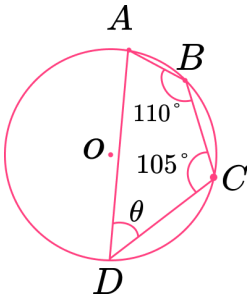
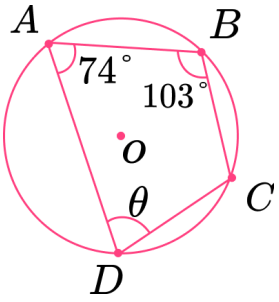


7)



8)



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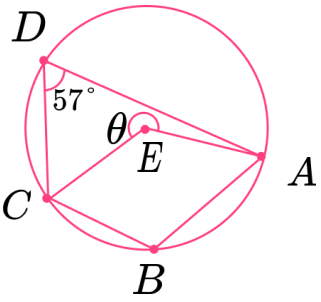

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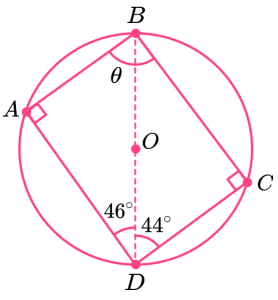

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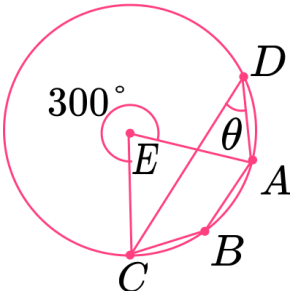

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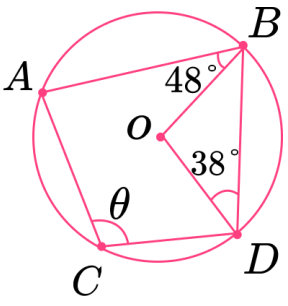
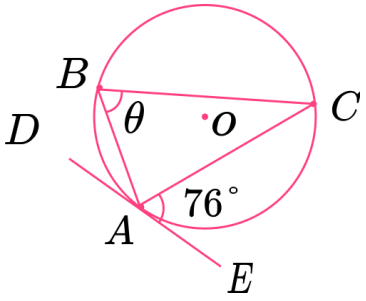


Circle Theorems - Worksheet

Group B - Alternate Segment Theorem, Angles in the Same Segment

Calculate the size of the missing angles marked θ

1)



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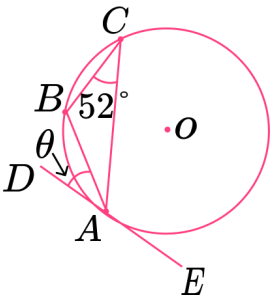
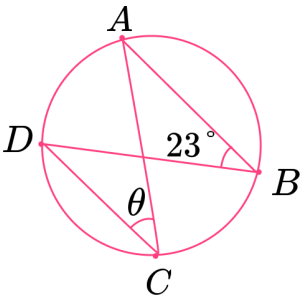


3)



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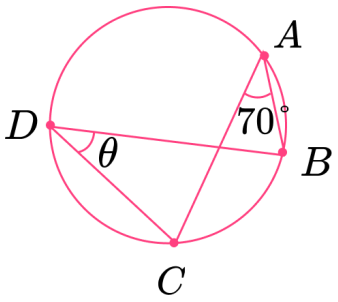

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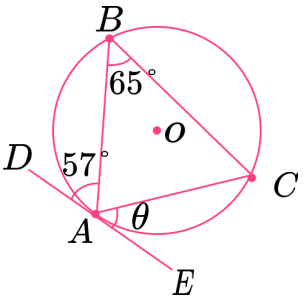
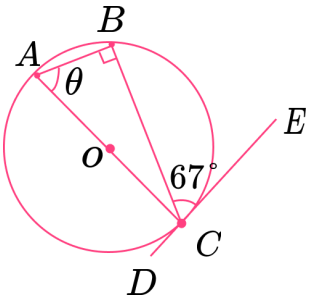
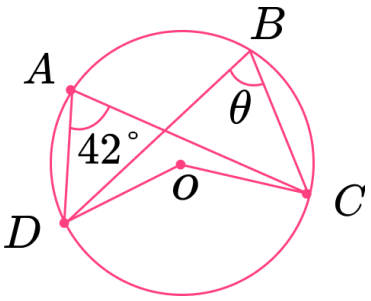


6)



7)



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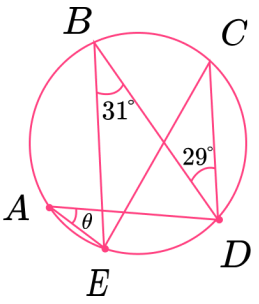

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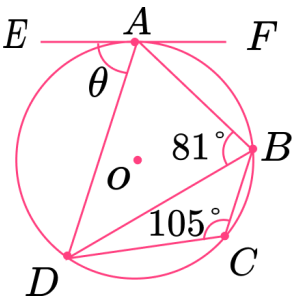

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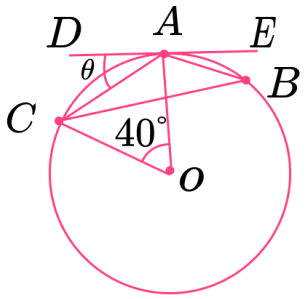
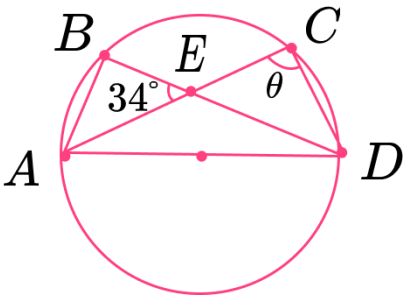


11)



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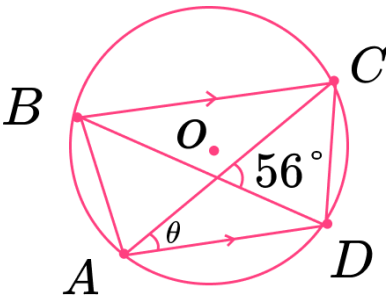
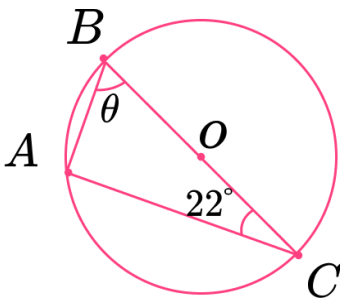


Circle Theorems - Worksheet

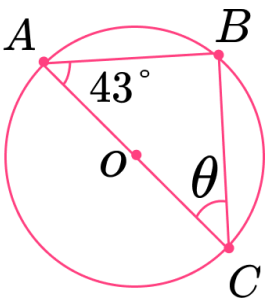
Group C - Angles in a Semicircle, Tangent of a Circle

Calculate the size of the missing angles marked θ

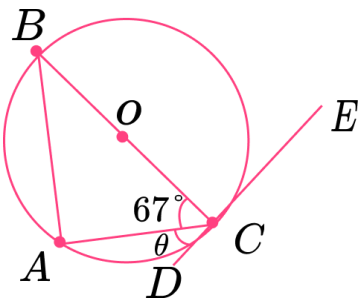
1)



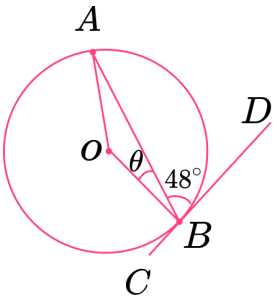
2)



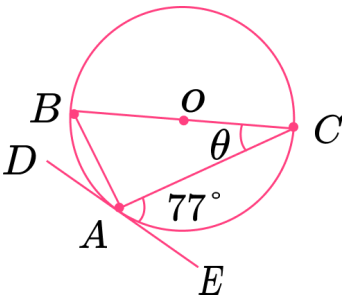
3)



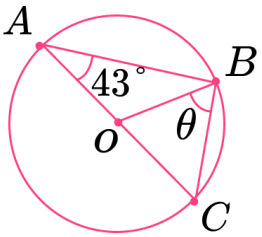
4)



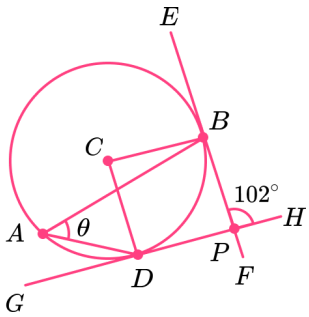
5)



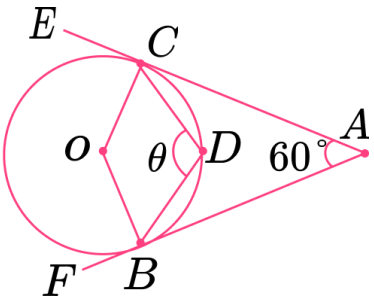
6)



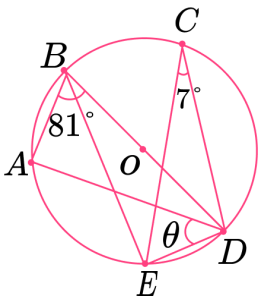
7)



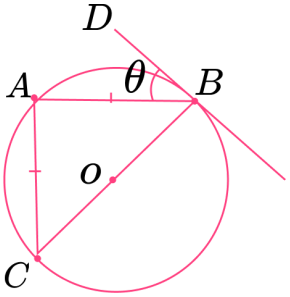
8)



9)



10)



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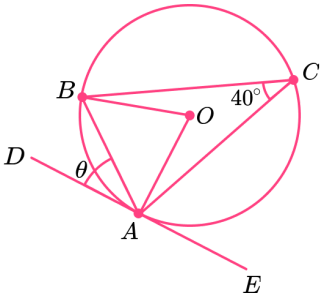

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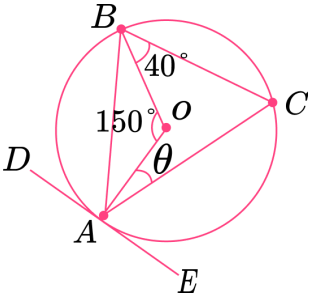
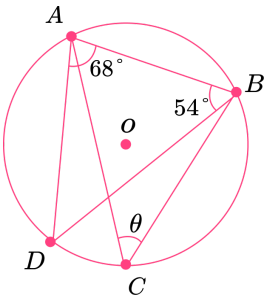


Circle Theorems - Worksheet

Applied

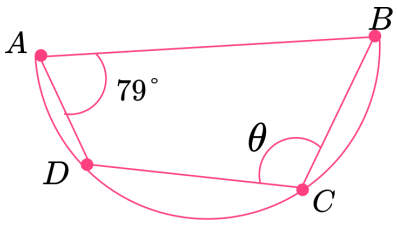
1) A, B, C and D are points on the circle with centre O.



(a) Calculate the size of angle ACB. Explain your answer.

(b) A new cord connects points C and D. Calculate the angle BCD.

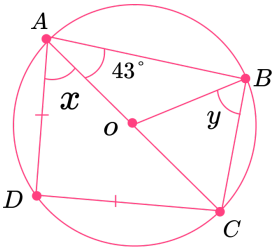
2) The diagram below shows a semicircle with the quadrilateral $ABCD$ inscribed inside.



(a) Calculate the size of angle BCD, labelled θ.

(b) A line connects A and C. What is the size of angle ACB. Explain your answer.

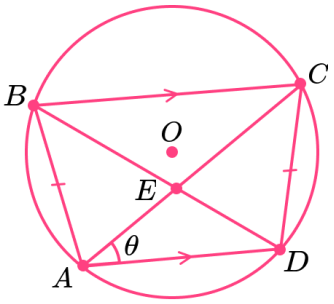
3) **(a)** Use the diagram below to calculate the value of x.



(b) Hence or otherwise, calculate the value of y.

Circle Theorems - Worksheet

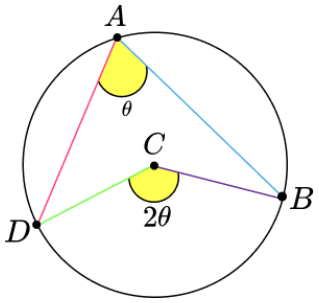
4) **(a)** BC and AD are parallel lines in the circle with centre O. Prove that AED is an isosceles triangle.



(b) Point F on the circumference lies between A and B. If angle $CAD = 52°$, what is the size of angle CFD. Explain your answer.

Circle Theorems - Exam Questions

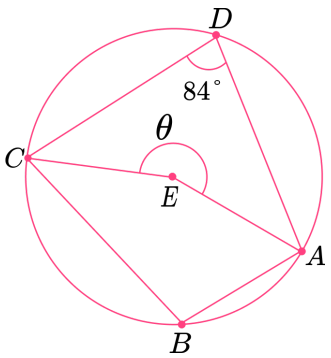
1) **(a)** Prove that the angle at the centre is twice the angle at the circumference.



.....

(5)

(b) Use this theorem to calculate the missing angle in the diagram:



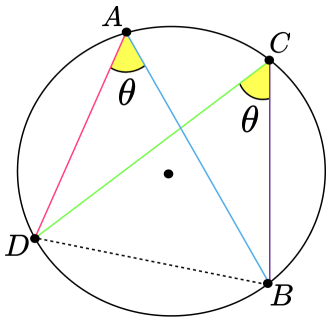
.....

(3)

(8 marks)

Circle Theorems - Exam Questions

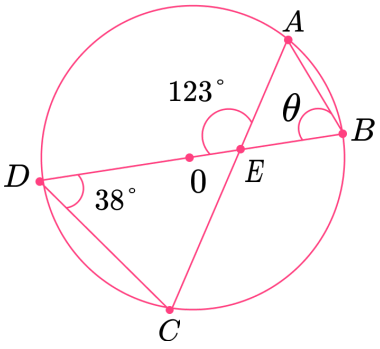
2) **(a)** Prove that angles in the same segment are equal.



.....

(3)

(b) Use this theorem to calculate the missing angle in the diagram:



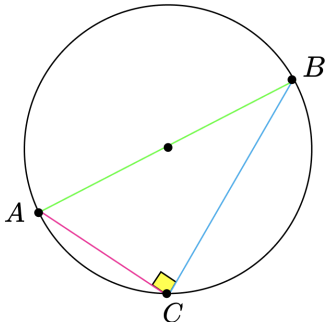
.....

(5)

(8 marks)

Circle Theorems - Exam Questions

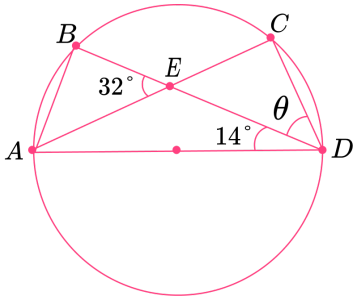
3) **(a)** Prove that the angle in a semicircle is 90 degrees.



.....

(4)

(b) Use this theorem to calculate the missing angle in the diagram:



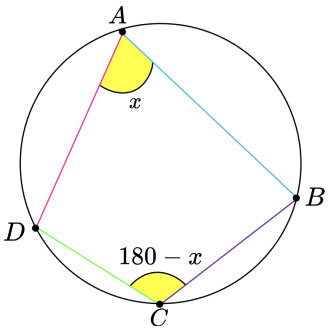
.....

(4)

(8 marks)

Circle Theorems - Exam Questions

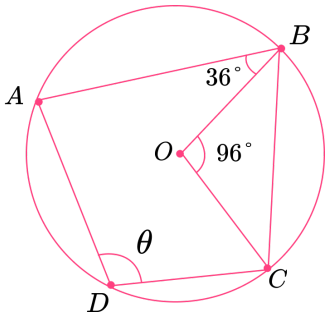
4) **(a)** Prove that opposite angles in a cyclic quadrilateral total 180 degrees.



.....

(4)

(b) Use this theorem to calculate the missing angle in the diagram:

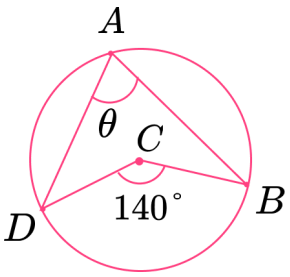
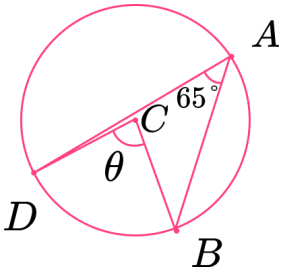
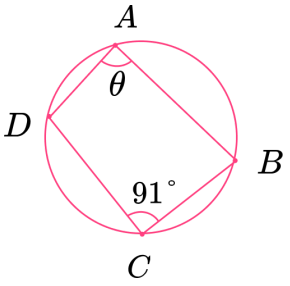
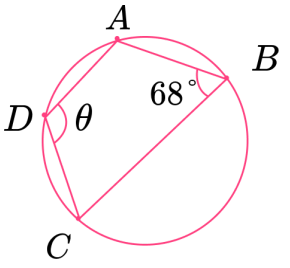
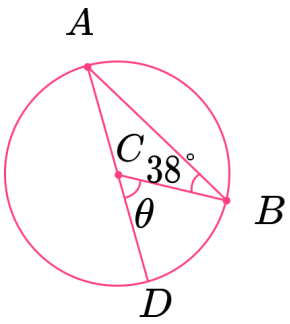


.....

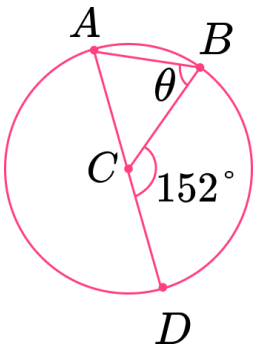
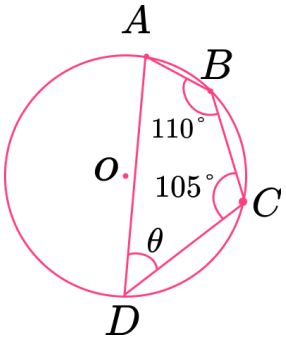
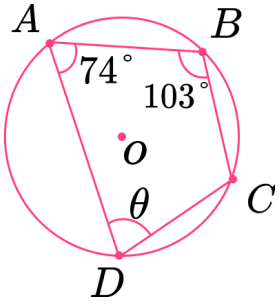
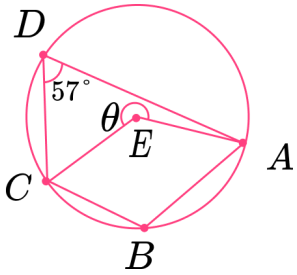
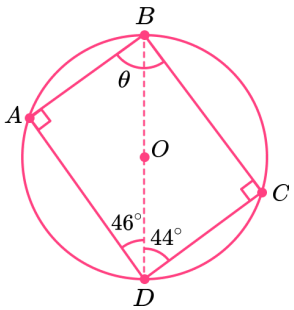
(3)

(7 marks)

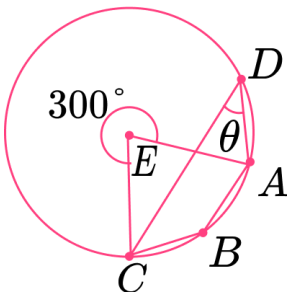
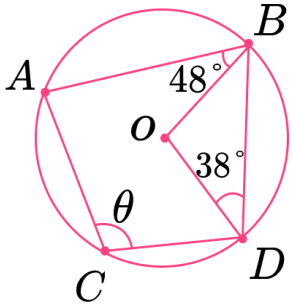
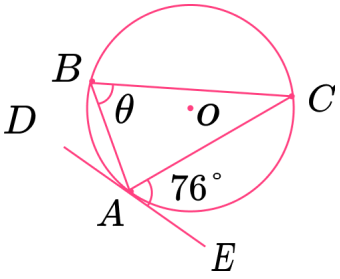
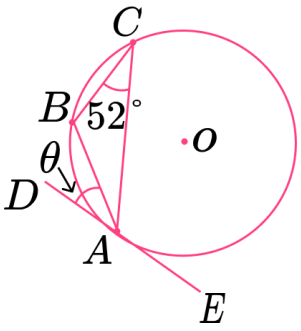
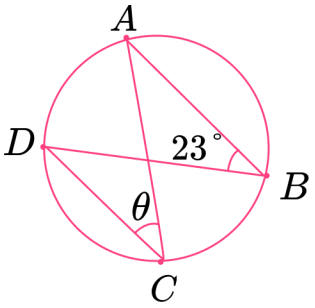
Circle Theorems - Answers

	Question	Answer
	Skill Questions	
Group A	Calculate the size of the missing angles marked θ	
	1) 	**1)** 70°
	2) 	**2)** 130°
	3) 	**3)** 89°
	4) 	**4)** 112°
	5) 	**5)** 76°

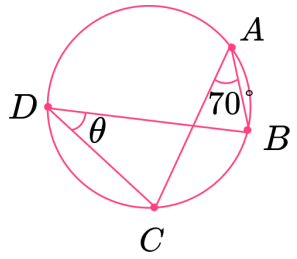
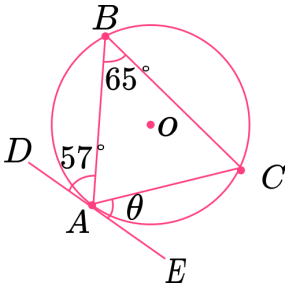
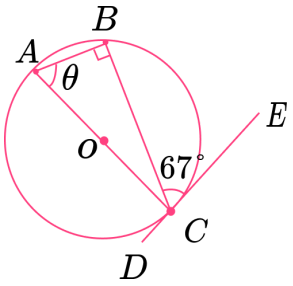
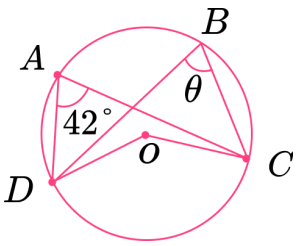
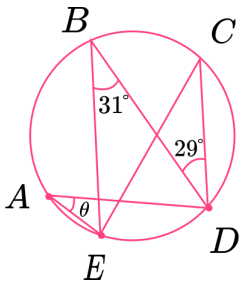
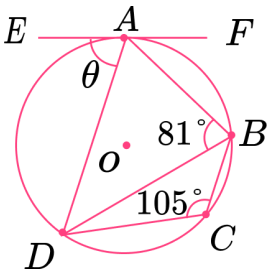
Circle Theorems - Answers

Group A contd	**6)** 	**6)** 76°
	7) 	**7)** 70°
	8) 	**8)** 77°
	9) 	**9)** 246°
	10) 	**10)** 90°

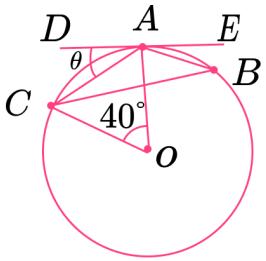
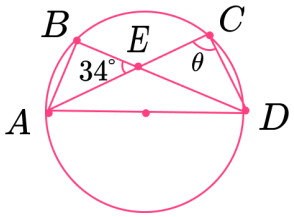
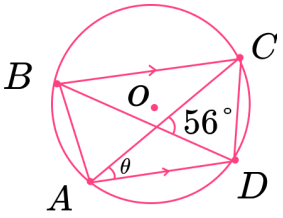
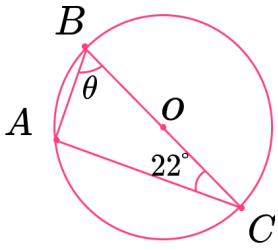
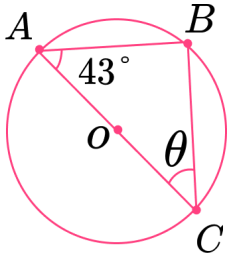
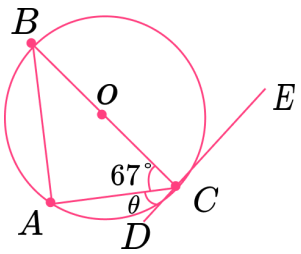
Circle Theorems - Answers

Group A contd	**11)**  **12)**  	**11)** 30° **12)** 94°
Group B	Calculate the size of the missing angles marked θ **1)**  **2)**  **3)**  	**1)** 76° **2)** 52° **3)** 23°

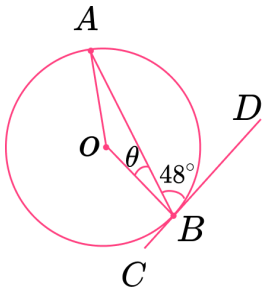
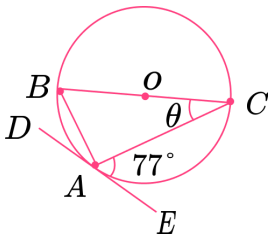
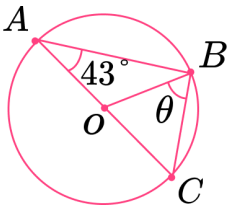
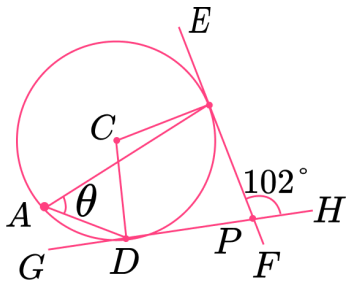
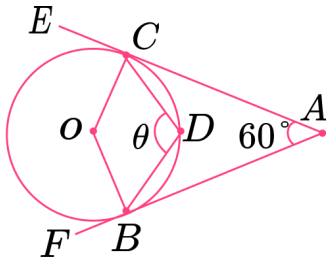
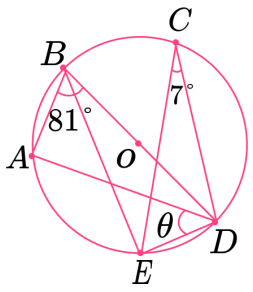
Circle Theorems - Answers

Group B contd	**4)** 	**4)** 70°
	5) 	**5)** 65°
	6) 	**6)** 67°
	7) 	**7)** 42°
	8) 	**8)** 31°
	9) 	**9)** 81°

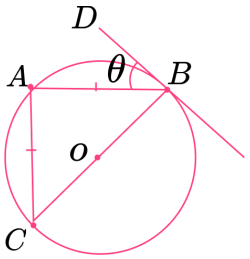
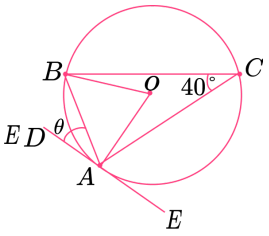
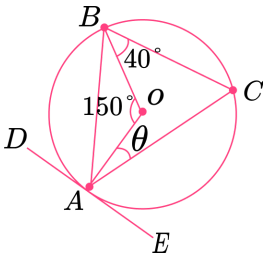
Circle Theorems - Answers

Group B contd	**10)** 	**10)** 20°
	11) 	**11)** 90°
	12) 	**12)** 28°
Group C	Calculate the size of the missing angles marked θ	
	1) 	**1)** 68°
	2) 	**2)** 47°
	3) 	**3)** 23°

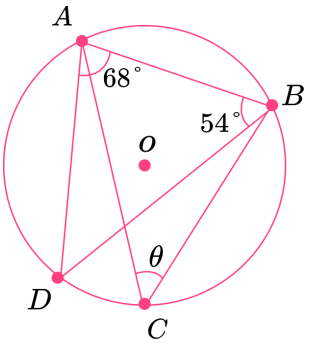
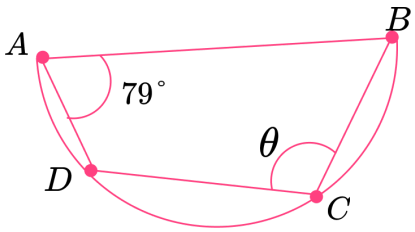
Circle Theorems - Answers

Group C contd	**4)** 	**4)** 42°
	5) 	**5)** 13°
	6) 	**6)** 47°
	7) 	**7)** 51°
	8) 	**8)** 120°
	9) 	**9)** 74°

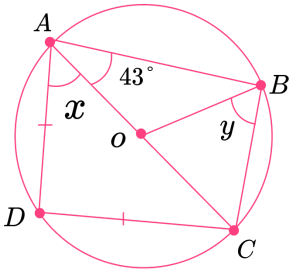
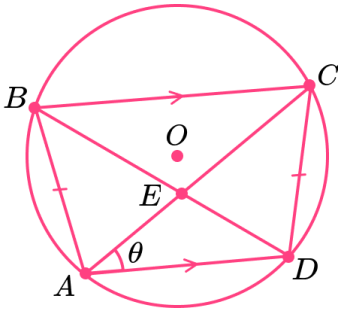
Circle Theorems - Answers

Group C contd	**10)** 	**10)** 45°
	11) 	**11)** 40°
	12) 	**12)** 35°

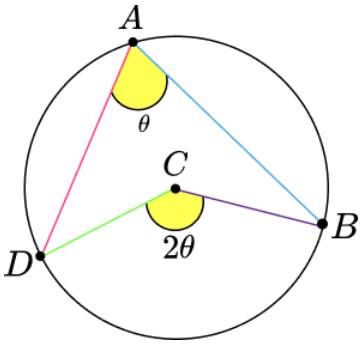
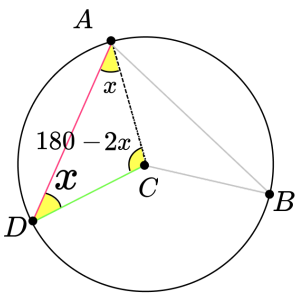
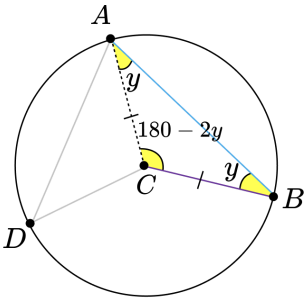
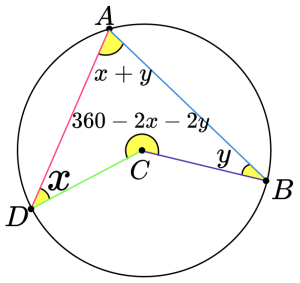
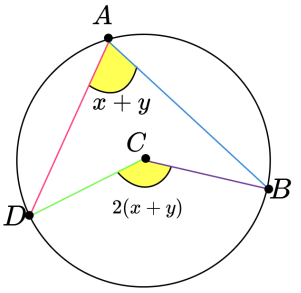
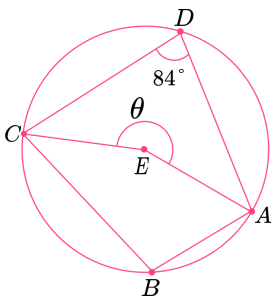
Circle Theorems- Answers

	Question	Answer
	Applied Questions	
1)	A, B, C and D are points on the circle with centre O.  	
	a) Calculate the size of angle ACB. Explain your answer.	**a)** $ADB = 180 - (68 + 54) = 58°$ Angles in a triangle total 180° $ACB = ADB = 58°$ Angles in the same segment are equal.
	b) A new chord connects points C and D. Calculate the angle BCD.	**b)** $180 - 68 = 112°$
2)	The diagram below shows a semicircle with the quadrilateral $ABCD$ inscribed inside.  	
	a) Calculate the size of angle BCD.	**a)** $BCD = 180 - 79 = 101°$
	b) A line connects A and C. What is the size of angle ACB. Explain your answer.	**b)** 90° as the angle in a semicircle is 90°.

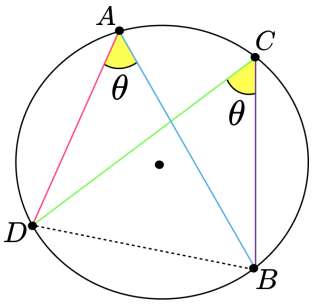
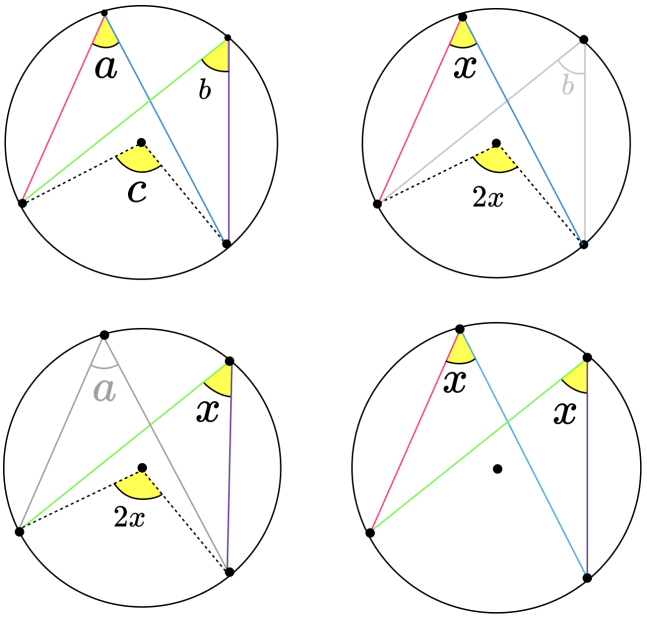
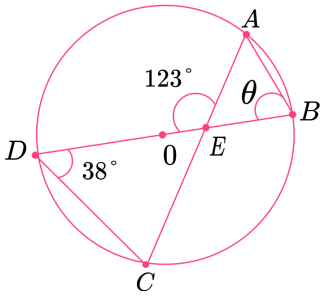
Circle Theorems- Answers

3)	**a)** Use the diagram below to calculate the value of x.  	**a)** As ACD is an isosceles triangle and angle $ADC = 90°$, $x = (180 - 90) \div 2 = 45°$
	b) Hence or otherwise, calculate the value of y.	**b)** $BOC = 2 \times 43 = 86°$ $180 - 86 = 94°$ $y = 94 \div 2 = 47°$
4)	**a)** BC and AD are parallel lines in the circle with centre O. Prove that AED is an isosceles triangle.  	**a)** $ABCD$ is an isosceles trapezium as AD and BC are parallel, and $AB = CD$. This means that the $AE = DE$ as the point E is the same distance along both diagonals BD and AC. $CAD = CBD$ as angles in the same segment are equal. $CAD = ACB$ as alternate angles are equal. Therefore $DAE = ADE = \theta$. $CED = 2CAD = 2\theta$ as the angle at the centre is twice the angle at the circumference. As AEC is a straight line, angle $AED = 180 - 2\theta$. The triangle has two equal sides and two equal angles and so it is an isosceles triangle. Note, if $\theta = 60^o$ then triangle AED is an equilateral triangle (a special type of isosceles triangle).
	b) Point F on the circumference lies between A and B. If angle $CAD = 52°$, what is the size of angle CFD. Explain your answer.	**b)** $CFD = 52°$ Angles in the same segment are equal

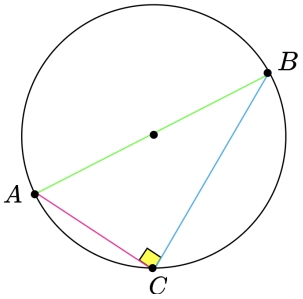
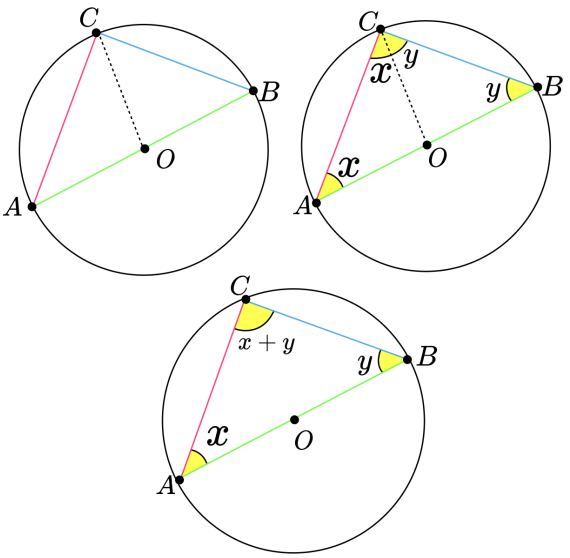
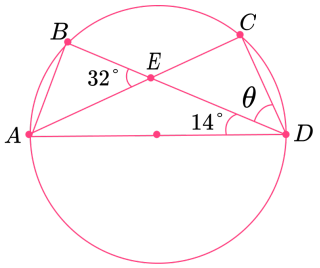
Circle Theorems - Mark Scheme

	Question	Answer	
	Exam Questions		
1) (a)	Prove that the angle at the centre is twice the angle at the circumference. 	(a)     ACD is an **isosceles triangle** so $CAD = ADC = x$. **(1)** Angle $ACD = 180 - 2x$ as **angles in a triangle total** 180°. **(1)** This is the same for triangle ABC using y instead of x. Adding the two angles at the centre, we get the reflex angle at $BCD = 360 - 2x - 2y$. **(1)** As **angles at a point total** 180°, the other angle at $BCD = 2x + 2y$ or $2(x + y)$. **(1)** As the angle at the circumference is equal to $x + y$ and the angle at the centre is equal to $2(x + y)$, we can state that **The angle at the centre is twice the angle at the circumference.** **(1)**
(b)	Use this theorem to calculate the missing angle in the diagram: 	(b) $CED = 84 \times 2 = 168$ **(1)** $\theta = 360 - 168 = 192°$ **(1)** Angles around a point total 360° **(1)**	

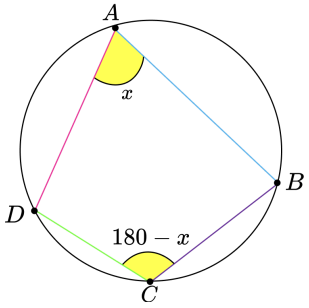
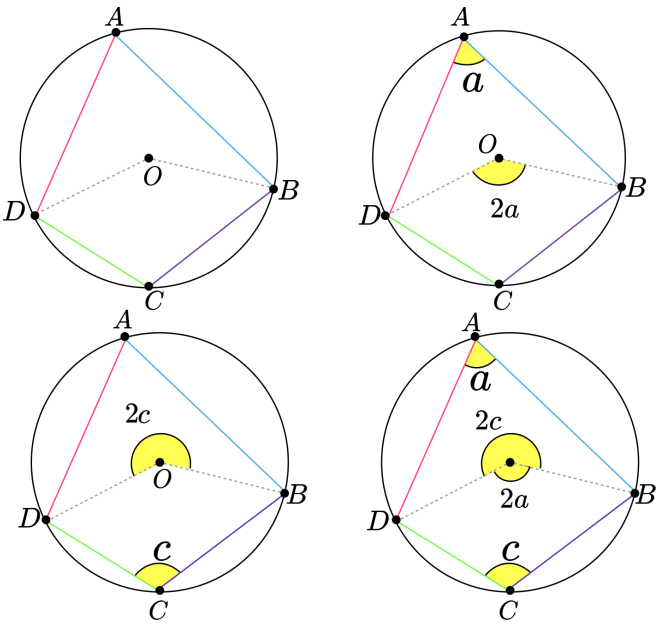
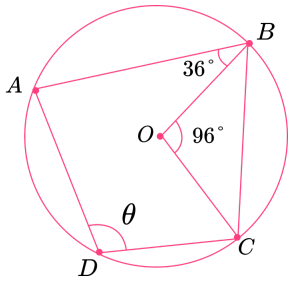
Circle Theorems - Mark Scheme

2) (a)	Prove that angles in the same segment are equal.  	**(a)**  As **the angle at the centre is twice the angle at the circumference,** angle c is twice the size of angle a. For the same reason, angle c is double the size of angle b. This means that $a = b$.	**(1)** **(1)** **(1)**
(b)	Use this theorem to calculate the missing angle in the diagram:  	**(b)** $CAB = 38°$ $AEB = 180 - 123 = 57°$ Reason: Angles on a straight line total 180° $\theta = 180 - (38 + 57) = 85°$ Reason: Angles in a triangle total 180°	**(1)** **(1)** **(1)** **(1)** **(1)**

Circle Theorems - Mark Scheme

3) (a)	Prove that the angle in a semicircle is 90 degrees.  	**(a)**  Splitting the triangle ABC into two **isosceles triangles**, we can state that angle $OAC = OCA = x$, and $OBC = OCB = y$. This means that angle $ACB = x + y$. As **angles in a triangle total** 180° $x + y + x + y = 180$ $2x + 2y = 180$ $x + y = 90°$.	**(1)** **(1)** **(1)** **(1)**
(b)	Use this theorem to calculate the missing angle in the diagram:  	**(b)** $ACD = 90°$ $CED = 32°$ $CDE = 180 - (90 + 32)$ $\theta = 58°$	**(1)** **(1)** **(1)** **(1)**

Circle Theorems - Mark Scheme

4) (a)	Prove that opposite angles in a cyclic quadrilateral total 180 degrees.  	**(a)**  As the **angle at the centre is twice the angle at the circumference** and $BAD = a$, angle $BOD = 2a$. For the same reason, The reflex angle at $BOD = 2c$. As **angles at a point total** 360° $2a + 2c = 360$ $a + c = 180°$	**(1)** **(1)** **(1)** **(1)** **(1)**
(b)	Use this theorem to calculate the missing angle in the diagram:  	**(b)** OBC is an isosceles triangle so $OBC = OCB = (180 - 96) \div 2 = 42°$ $ABC = 42 + 36 = 78°$ $ADC = 180 - 78 = 102°$	**(1)** **(1)** **(1)**