Iteration - Worksheet

Skill

Group A - Iteration

Calculate the value of x_3 when:

1) $x_1 = 5$ and $x_{n+1} = 3(x_n - 3)$

2) $x_1 = 4$ and $x_{n+1} = 5(x_n - 5)$

3) $x_1 = 3$ and $x_{n+1} = 3x_n - 3$

4) $x_1 = 4$ and $x_{n+1} = 3(x_n + 5)$

5) $x_1 = 2$ and $x_{n+1} = 6x_n - 3$

6) $x_1 = 2$ and $x_{n+1} = 6x_n - 4$

7) $x_1 = 6$ and $x_{n+1} = 2(x_n + 3)$

8) $x_1 = 1$ and $x_{n+1} = 5x_n + 2$

9) $x_1 = 1$ and $x_{n+1} = 3(x_n - 1)$

10) $x_1 = 2$ and $x_{n+1} = 7x_n - 2$

11) $x_1 = 1$ and $x_{n+1} = 3(x_n + 6)$

12) $x_1 = 5$ and $x_{n+1} = 3x_n - 3$

Group B - Iteration (including powers and roots)

Find x_3 using the information below. Give your answers to 3 d.p:

1) $x_{n+1} = \frac{3}{x_n^2+1}$ and $x_0 = 1$

2) $x_{n+1} = \frac{1}{3x_n^2+1}$ and $x_0 = 0$

3) $x_{n+1} = \frac{2-3x_n^3}{3}$ and $x_0 = 0$

4) $x_{n+1} = \sqrt{\frac{1-2x_n^3}{3}}$ and $x_0 = 0$

5) $x_{n+1} = \frac{3-3x_n^3}{4}$ and $x_0 = 0$

6) $x_{n+1} = \frac{2}{3x_n^2+3}$ and $x_0 = 0$

7) $x_{n+1} = \sqrt{\frac{2-3x_n^3}{4}}$ and $x_0 = 0$

8) $x_{n+1} = \frac{2}{4x_n^2+4}$ and $x_0 = 0$

9) $x_{n+1} = \sqrt{\frac{2-2x_n^3}{5}}$ and $x_0 = 0$

10) $x_{n+1} = \frac{1}{2x_n^2+3}$ and $x_0 = 0$

11) $x_{n+1} = \sqrt{\frac{1-x_n^3}{2}}$ and $x_0 = 0$

12) $x_{n+1} = \sqrt{\frac{-3-x_n^3}{-3}}$ and $x_0 = 1$

Iteration - Worksheet

Group C - Iteration (working backwards)

Calculate:

1) The value of x_1 when $x_2 = 0$ and $x_{n+1} = 2x_n - 4$

2) The value of x_1 when $x_2 = 7$ and $x_{n+1} = 3x_n - 5$

3) The value of x_1 when$x_2 = 6$ and $x_{n+1} = 2x_n - 6$

4) The value of x_1 when$x_2 = 17$ and $x_{n+1} = 5x_n - 3$

5) The value of x_1 when $x_3 = 54$ and $x_{n+1} = 2(x_n + 5)$

6) The value of x_1 when $x_3 = -24$ and $x_{n+1} = 4(x_n - 2)$

7) The value of x_1 when$x_3 = 38$ and $x_{n+1} = 3x_n - 4$

8) The value of x_1 when $x_3 = 63$ and $x_{n+1} = 3(x_n + 3)$

9) The value of x_1 when $x_3 = 4$ and $x_{n+1} = 4(x_n - 3)$

10) The value of x_1 when $x_3 = -36$ and $x_{n+1} = 4(x_n - 5)$

11) The value of x_1 when $x_3 = 19$ and $x_{n+1} = 2x_n - 3$

12) The value of x_1 when $x_3 = -20$ and $x_{n+1} = 2(x_n - 6)$

Iteration - Worksheet

Applied

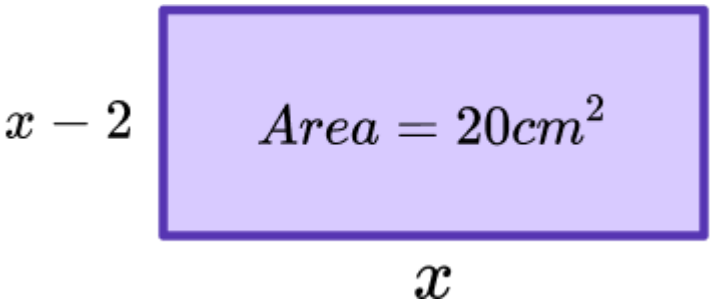
1) **(a)** Show that the equation $x^3 + 2x = 3$ can be rearranged to give $x = \frac{3}{2} - \frac{x^3}{2}$

(b) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{3}{2} - \frac{x_n^3}{2}$ twice, to find an estimate to the solution of $x^3 + 2x = 3$

2) The annual bird population in Indonesia can be modelled using the iterative formula $P_{n+1} = 1.5(P_n - 600)$

The population this year is 5746
Find the population in two years time. Give your answer to 3 significant figures.

3) Below is a rectangle.



(a) Form a quadratic equation to represent the area of the rectangle. Give your answer in the form $ax^2 + bx + c = 0$

(b) Show that the quadratic equation can be rearranged to give $x = \sqrt{2x + 20}$

(c) Starting with $x_0 = 5$, use the iteration formula $x_{n+1} = \sqrt{2x_n + 20}$ three times, to find an estimate for the length x of the rectangle.

4) The annual spider population in the Amazon rainforest can be modelled using the iterative formula $P_{n+1} = 1.4(P_n - 500)$

The population this year is 1437
Find the population in two years time. Give your answer to the nearest whole number.

Iteration - Exam Questions

1) An approximate solution to an equation is found using this iterative process:

$x_{n+1} = \sqrt{x_n + 10}$ and $x_1 = 3$

Work out the values of x_2 and x_3

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(3 marks)

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2) Using $x_{n+1} = 9 - \frac{5}{x_n^2}$ with $x_0 = 1$ find the values of:

(a) x_1

.....

(1)

(b) x_2

.....

(1)

(c) x_3

.....

(1)

(3 marks)

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3) **(a)** Show that the equation $x^3 + 5x = 2$ can be rearranged to give $x = \frac{2}{5} - \frac{x^3}{5}$

.....

(2)

(b) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{2}{5} - \frac{x_n^3}{5}$ twice, to find an estimate to the solution of $x^3 + 5x = 2$

.....

(2)

(4 marks)

Iteration - Answers

	Question	Answer
	Skill Questions	
Group A	Calculate: **1)** The value of x_3 when $x_1 = 5$ and $x_{n+1} = 3(x_n - 3)$ **2)** The value of x_3 when $x_1 = 4$ and $x_{n+1} = 5(x_n - 5)$ **3)** The value of x_3 when $x_1 = 3$ and $x_{n+1} = 3x_n - 3$ **4)** The value of x_3 when $x_1 = 4$ and $x_{n+1} = 3(x_n + 5)$ **5)** The value of x_3 when $x_1 = 2$ and $x_{n+1} = 6x_n - 3$ **6)** The value of x_3 when $x_1 = 2$ and $x_{n+1} = 6x_n - 4$ **7)** The value of x_3 when $x_1 = 6$ and $x_{n+1} = 2(x_n + 3)$ **8)** The value of x_3 when $x_1 = 1$ and $x_{n+1} = 5x_n + 2$ **9)** The value of x_3 when $x_1 = 1$ and $x_{n+1} = 3(x_n - 1)$ **10)** The value of x_3 when $x_1 = 2$ and $x_{n+1} = 7x_n - 2$ **11)** The value of x_3 when $x_1 = 1$ and $x_{n+1} = 3(x_n + 6)$ **12)** The value of x_3 when $x_1 = 5$ and $x_{n+1} = 3x_n - 3$	**1)** 9 **2)** -50 **3)** 15 **4)** 96 **5)** 51 **6)** 44 **7)** 42 **8)** 37 **9)** -3 **10)** 82 **11)** 81 **12)** 33

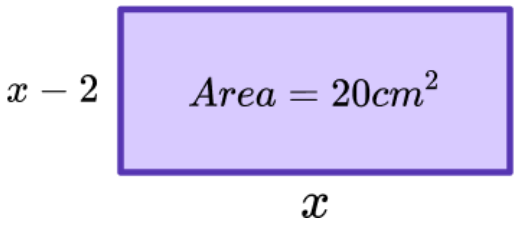
Iteration - Answers

Group B	Find x_3 using the information below. Give your answers to 3dp:	
	1) $x_{n+1} = \frac{3}{x_n^2+1}$ and $x_0 = 1$	**1)** 1.620
	2) $x_{n+1} = \frac{1}{3x_n^2+1}$ and $x_0 = 0$	**2)** 0.842
	3) $x_{n+1} = \frac{2-3x_n^3}{3}$ and $x_0 = 0$	**3)** 0.616
	4) $x_{n+1} = \sqrt{\frac{1-2x_n^3}{3}}$ and $x_0 = 0$	**4)** 0.521
	5) $x_{n+1} = \frac{3-3x_n^3}{4}$ and $x_0 = 0$	**5)** 0.689
	6) $x_{n+1} = \frac{2}{3x_n^2+3}$ and $x_0 = 0$	**6)** 0.550
	7) $x_{n+1} = \sqrt{\frac{2-3x_n^3}{4}}$ and $x_0 = 0$	**7)** 0.644
	8) $x_{n+1} = \frac{2}{4x_n^2+4}$ and $x_0 = 0$	**8)** 0.431
	9) $x_{n+1} = \sqrt{\frac{2-2x_n^3}{5}}$ and $x_0 = 0$	**9)** 0.579
	10) $x_{n+1} = \frac{1}{2x_n^2+3}$ and $x_0 = 0$	**10)** 0.313
	11) $x_{n+1} = \sqrt{\frac{1-x_n^3}{2}}$ and $x_0 = 0$	**11)** 0.639
	12) $x_{n+1} = \sqrt{\frac{-3-x_n^3}{-3}}$ and $x_0 = 1$	**12)** 1.273

Iteration - Answers

Group C	Work out: **1)** The value of x_1 when $x_2 = 0$ and $x_{n+1} = 2x_n - 4$. **2)** The value of x_1 when $x_2 = 7$ and $x_{n+1} = 3x_n - 5$. **3)** The value of x_1 when $x_2 = 6$ and $x_{n+1} = 2x_n - 6$ **4)** The value of x_1 when $x_2 = 17$ and $x_{n+1} = 5x_n - 3$ **5)** The value of x_1 when $x_3 = 54$ and $x_{n+1} = 2(x_n + 5)$ **6)** The value of x_1 when $x_3 = -24$ and $x_{n+1} = 4(x_n - 2)$ **7)** The value of x_1 when $x_3 = 38$ and $x_{n+1} = 3x_n - 4$ **8)** The value of x_1 when $x_3 = 63$ and $x_{n+1} = 3(x_n + 3)$ **9)** The value of x_1 when $x_3 = 4$ and $x_{n+1} = 4(x_n - 3)$ **10)** The value of x_1 when $x_3 = -36$ and $x_{n+1} = 4(x_n - 5)$ **11)** The value of x_1 when $x_3 = 19$ and $x_{n+1} = 2x_n - 3$ **12)** The value of x_1 when $x_3 = -20$ and $x_{n+1} = 2(x_n - 6)$	**1)** 2 **2)** 4 **3)** 6 **4)** 4 **5)** 6 **6)** 1 **7)** 6 **8)** 3 **9)** 4 **10)** 4 **11)** 7 **12)** 4

Iteration - Answers

	Question	Answer
	Applied Questions	
1)	**a)** Show that the equation $x^3 + 2x = 3$ can be rearranged to give $x = \frac{3}{2} - \frac{x^3}{2}$ **b)** Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{3}{2} - \frac{x_n^3}{2}$ twice, to find an estimate to the solution of $x^3 + 2x = 3$	**a)** $x^3 + 2x = 3$ $-x^3$ $-x^3$ $2x = 3 - x^3$ $\div 2$ $\div 2$ $x = \frac{3}{2} - \frac{x^3}{2}$ **b)** $x_1 = \frac{3}{2}$ $x_2 = -\frac{3}{16}$
2)	The annual bird population in Indonesia can be modelled using the iterative formula $P_{n+1} = 1.5(P_n - 600)$ The population this year is 5746 Find the population in two years time. Give your answer to 3 significant figures.	$x_2 = 10678.5$ Population in 2017 $= 10700$
3)	**a)** Below is a rectangle.  Form a quadratic equation to represent the area of the rectangle. Give your answer in the form $ax^2 + bx + c = 0$ **b)** Show that the quadratic equation can be rearranged to give $x = \sqrt{2x + 20}$ **c)** Starting with $x_0 = 5$, use the iteration formula $x_{n+1} = \sqrt{2x_n + 20}$ three times, to find an estimate for the length x of the rectangle.	**a)** $x(x - 2) = 20$ $x^2 - 2x = 20$ $x^2 - 2x - 20 = 0$ **b)** $x^2 - 2x - 20 = 0$ $+20$ $+20$ $x^2 - 2x = 20$ $+2x$ $+2x$ $x^2 = 2x + 20$ $\sqrt{}$ $\sqrt{}$ $x = \sqrt{2x + 20}$ **c)** $x_1 = 5.4772...$ $x_2 = 5.5636...$ $x_3 = 5.579\ (3dp)$

Iteration - Answers

4)	The annual spider population in the Amazon rainforest can be modelled using the iterative formula $P_{n+1} = 1.4(P_n - 500)$. The population this year is 1437 Find the population in two years time. Give your answer to the nearest whole number.	$x_2 = 1136.52$ Population = 1137

Iteration - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	An approximate solution to an equation is found using this iterative process $x_{n+1} = \sqrt{x_n + 10}$ and $x_1 = 3$ Work out the values of x_2 and x_3	$x_2 = \sqrt{3 + 10}$ **(1)** $x_2 = \sqrt{13} = 3.605551...$ **(1)** $x_3 = 3.68857...$ **(1)**	**(3)**
2)	Using $x_{n+1} = 9 - \frac{5}{x_n^2}$ with $x_0 = 1$ find the values of		**(1)**
(a)	x_1	**(a)** $x_1 = 4$	
(b)	x_2	**(b)** $x_2 = \frac{139}{16} = 8.6875$	**(1)**
(c)	x_3	**(c)** $x_3 = 8.93375...$	**(1)**
3) (a)	Show that the equation $x^3 + 5x = 2$ can be rearranged to give $x = \frac{2}{5} - \frac{x^3}{5}$	**(a)** $x^3 + 5x = 2$ $-x^3$ $-x^3$ $5x = 2 - x^3$ $\div 5$ $\div 5$ $x = \frac{2}{5} - \frac{x^3}{5}$ For correct first step **(1)** For correct second step **(1)**	**(2)**
(b)	Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{2}{5} - \frac{x_n^3}{5}$ twice, to find an estimate to the solution of $x^3 + 5x = 2$	**(b)** $x_1 = \frac{2}{5}$ **(1)** $x_2 = 0.3872$ **(1)**	**(2)**