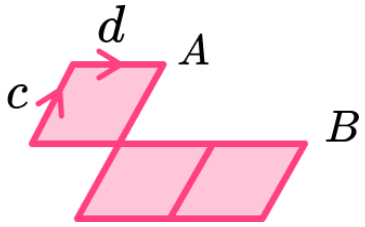
Vector problems - Worksheet

Skill

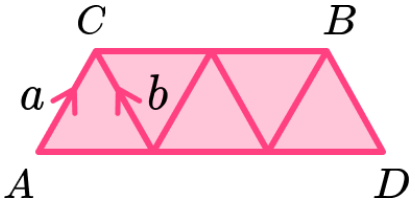
Group A - Vector problems including parallel and extended lines

In each question find the vector $\overrightarrow{AB}$. All diagrams are not to scale:

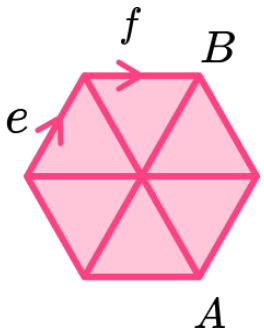
1)



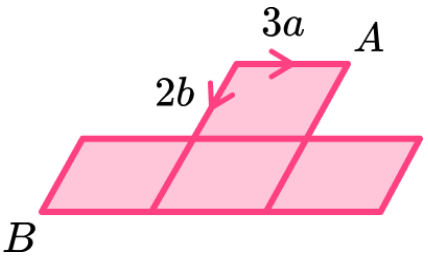
2)



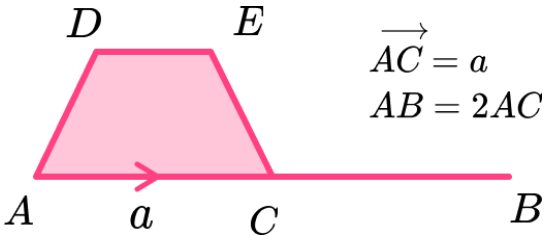
3)



4)



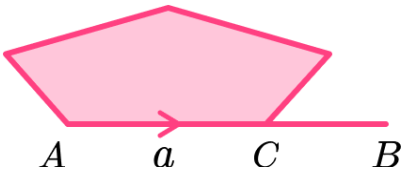
5)



$\overrightarrow{AC} = a$

$AB = 2AC$

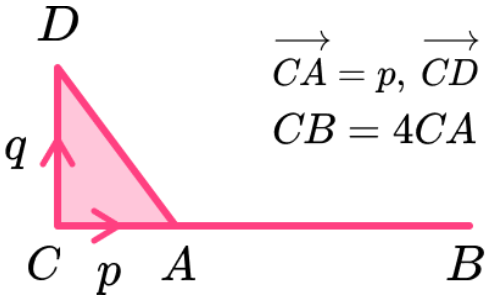
6)



$\overrightarrow{AC} = a$

$CB = \frac{1}{2}AC$

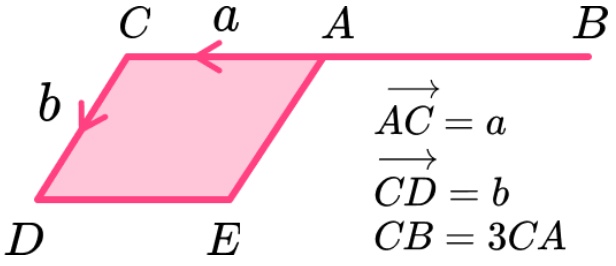
7)



$\overrightarrow{CA} = p,\ \overrightarrow{CD} = q$

$CB = 4CA$

8)



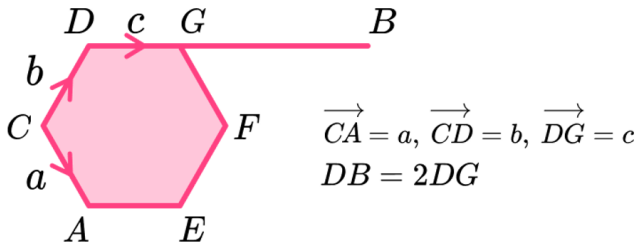
$\overrightarrow{AC} = a$

$\overrightarrow{CD} = b$

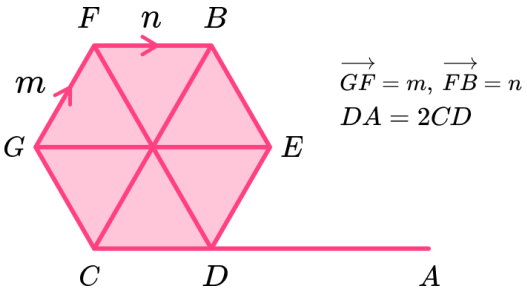
$CB = 3CA$

Vector problems - Worksheet

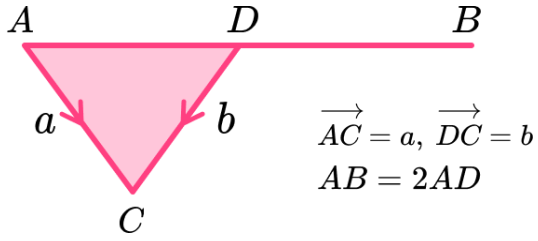
9)



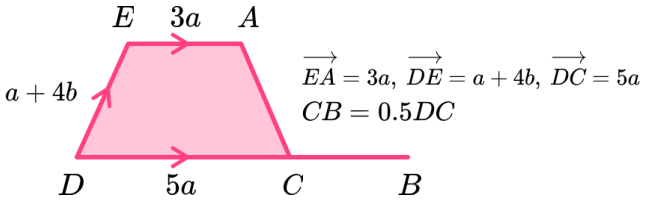
10)



11)



12)

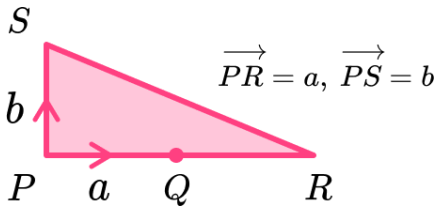


Group B - Vector problems including midpoints

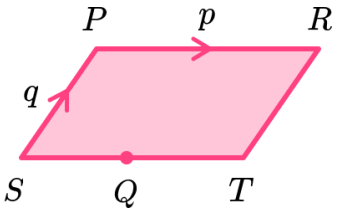
In questions 1-8 find the vector $\overrightarrow{PQ}$. In questions 9-12 find the vector $\overrightarrow{MQ}$.

Both M and Q are midpoints. All diagrams are not to scale:

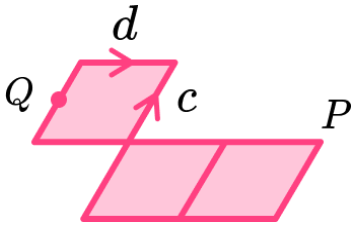
1)



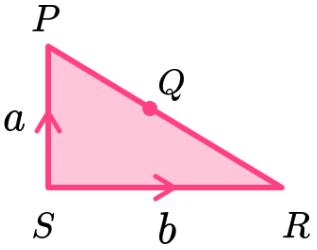
2)



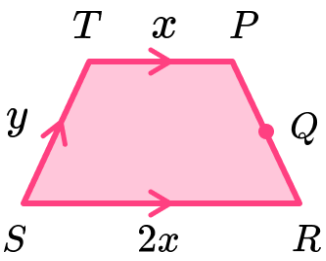
3)



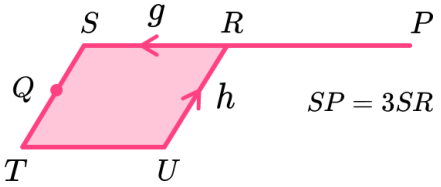
4)



5)

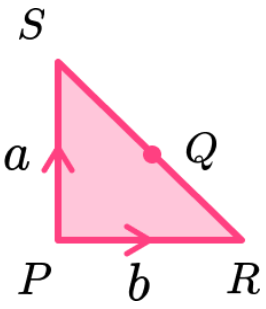


6)

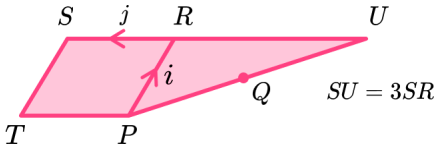


Vector problems - Worksheet

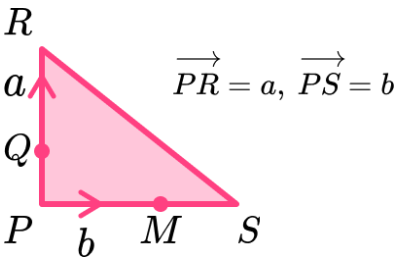
7)



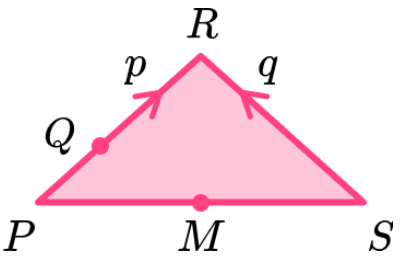
8)



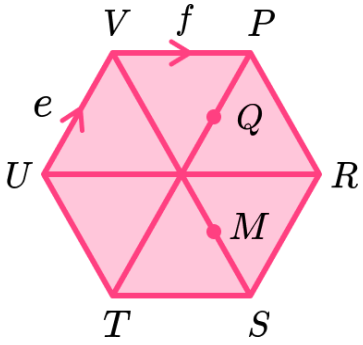
9)



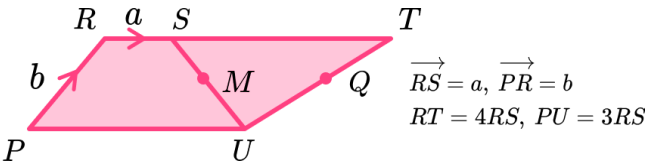
10)



11)



12)

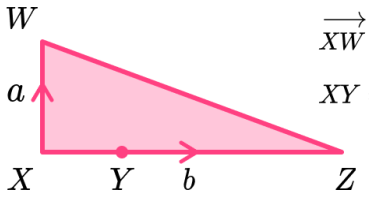


Vector problems - Worksheet

Group C - Vector problems including fractions and ratios

In each question find the vector $\overrightarrow{XY}$. All diagrams are not to scale:

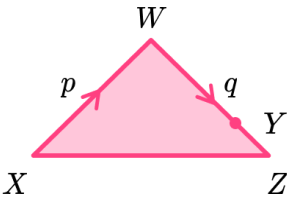
1)



$\overrightarrow{XW} = a$, $\overrightarrow{XZ} = b$

$XY = \frac{1}{3}XZ$

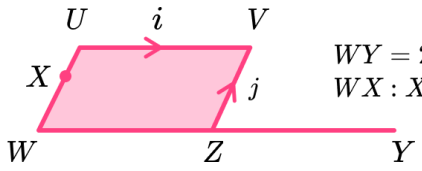
2)



$\overrightarrow{XW} = p$, $\overrightarrow{WZ} = q$

$WY : YZ = 3 : 1$

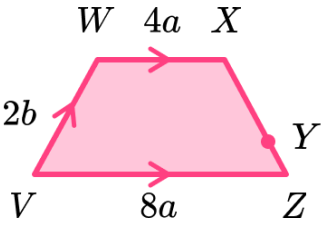
3) UVZW is a parallelogram



$WY = 2WZ$

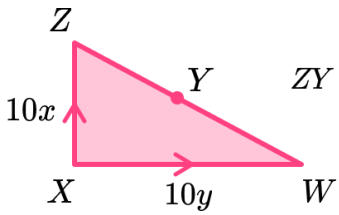
$WX : XU = 2 : 1$

4)



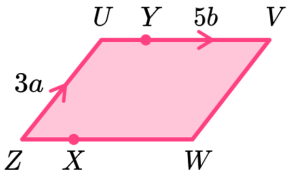
$YZ = \frac{1}{4}XZ$

5)



$ZY : YW = 2 : 3$

6) UVWZ is a parallelogram

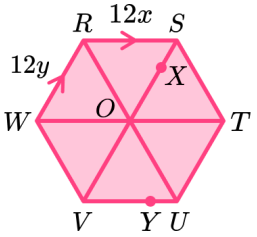


$\overrightarrow{ZU} = 3a$, $\overrightarrow{UV} = 5b$

$ZX = \frac{2}{3}XW$

$UY : YV = 1 : 4$

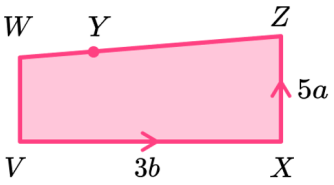
7)



$SX = \frac{1}{4}SO$

$VY : YU = 3 : 1$

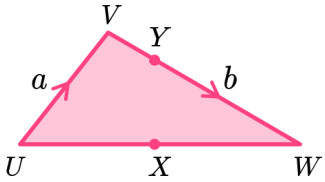
8)



$VW = \frac{1}{2}XZ$

$WY : YZ = 1 : 2$

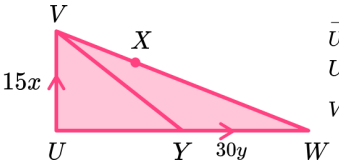
9)



$X = midpoint\ UW$

$VY = \frac{1}{4}VW$

10)

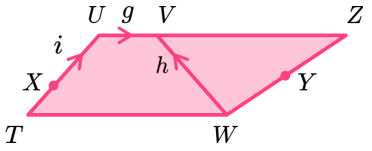


$\overrightarrow{UV} = 15x$, $\overrightarrow{UW} = 30y$

$UY : YW = 2 : 3$

$VX = \frac{1}{3}VW$

11)



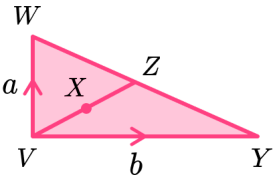
$\overrightarrow{UV} = g$, $\overrightarrow{VW} = h$, $\overrightarrow{TU} = i$

$UZ = 4UV$

$TX : XU = 1 : 3$

$Y = midpoint\ WZ$

12)



$WZ : ZY = 3 : 2$

$X = midpoint\ VZ$

Vector problems - Worksheet

Applied

1) Point D is located using the vector $\overrightarrow{CD} = \boldsymbol{a} + 2\boldsymbol{b}$.
Point E is located using the vector $\overrightarrow{AE} = -\boldsymbol{a}$.

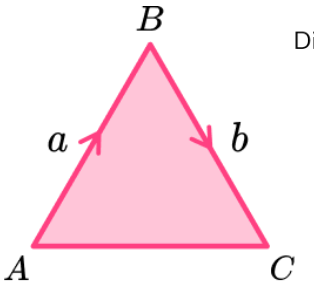


Diagram **not** to scale

(a) Make a copy of the diagram, showing the location of the points D and E.

(b) Hence or otherwise find the vector $\overrightarrow{DE}$.

2) In the diagram below, $\overrightarrow{AB} = \boldsymbol{a}$, $\overrightarrow{DA} = \boldsymbol{b}$ and $\overrightarrow{DC} = 3\overrightarrow{AB}$.
The line AB is extended so that $\overrightarrow{AE} = 2\overrightarrow{AB}$.
M is the midpoint of BC.
N is the point such that $DN: NC = 2: 1$.

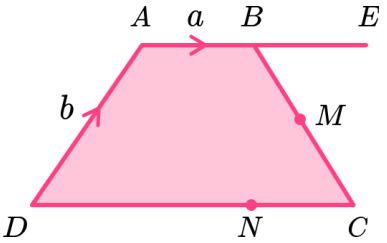


Diagram **not** to scale

(a) **i)** Find the vector $\overrightarrow{CD}$.
ii) Find the vector $\overrightarrow{CB}$.
iii) Find the vector $\overrightarrow{NM}$.

(b) Show that NME is a straight line.

Vector problems - Worksheet

3) In the diagram below, $\overrightarrow{AD} = \boldsymbol{x}$ and $\overrightarrow{DC} = \boldsymbol{y}$.
E is the midpoint of AC.

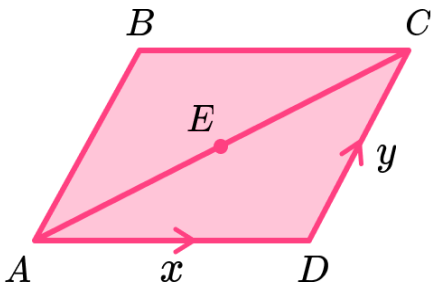


Diagram **not** to scale

(a) Find the vector $\overrightarrow{AE}$.

(b) Show that E is the midpoint of BD.

4) In the diagram below, $\overrightarrow{BA} = 3\boldsymbol{a}$, $\overrightarrow{BC} = 3\boldsymbol{b}$ and $\overrightarrow{AF} = 3\overrightarrow{AD}$.
E is a point such that $\overrightarrow{ED} = n\boldsymbol{a}$.

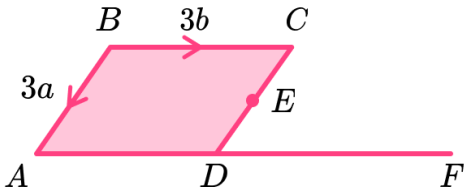


Diagram **not** to scale

Given that BEF is a straight line, find the value of n.

Vector problems - Exam Questions

1) $\overrightarrow{AB} = 2\boldsymbol{x} + 4\boldsymbol{y}$, $\overrightarrow{AC} = 8\boldsymbol{y}$.

D is the midpoint of AB.

The line AC is extended so that $\overrightarrow{AE} = 1.5\overrightarrow{AC}$.

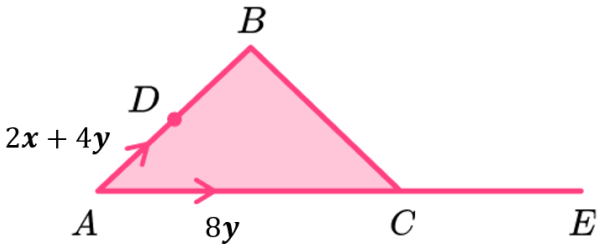


Diagram **not** to scale

(a) Find the vector $\overrightarrow{BC}$.

.....

(1)

(b) Find the vector $\overrightarrow{DE}$.

.....

(2)

(3 marks)

Vector problems - Exam Questions

2) $\overrightarrow{AB} = 10\boldsymbol{a}$, $\overrightarrow{BC} = 5\boldsymbol{b}$.

The point E lies on the line AC such that $AE : EC = 3 : 2$.

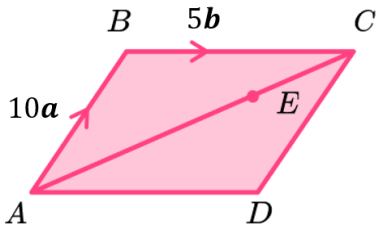


Diagram **not** to scale

(a) Find the vector $\overrightarrow{AC}$.

.....

(1)

(b) Show that $\overrightarrow{AE} = k(2\boldsymbol{a} + \boldsymbol{b})$ and hence determine the value of k.

.....

(2)

(3 marks)

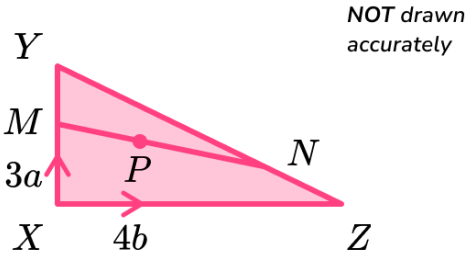
Vector problems - Exam Questions

3) $\overrightarrow{XY} = 3\boldsymbol{a}$, $\overrightarrow{XZ} = 4\boldsymbol{b}$

$\overrightarrow{MY} = \frac{1}{3}\overrightarrow{XY}$

$ZN : NY = 1 : 3$

P is the midpoint of $\overrightarrow{MN}$.



(a) Find the vector $\overrightarrow{MN}$.

.....

(3)

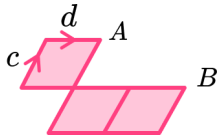
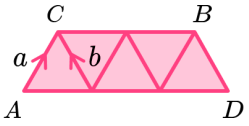
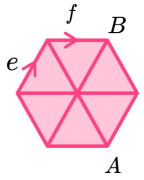
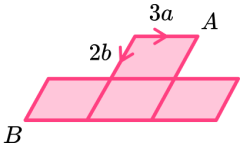
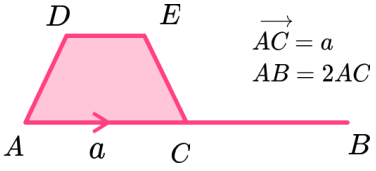
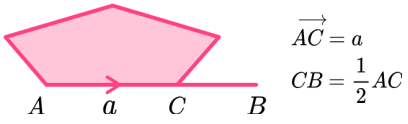
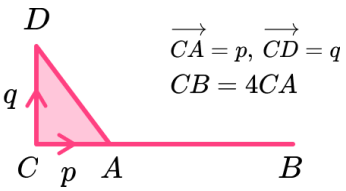
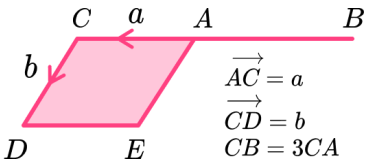
(b) Find the vector $\overrightarrow{XP}$.

.....

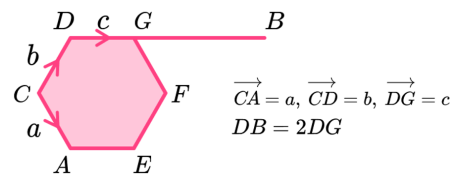
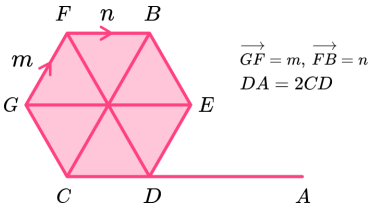
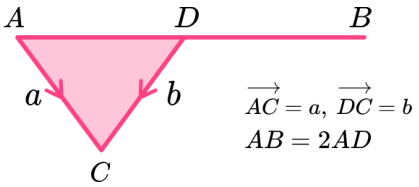
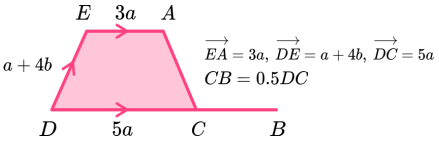
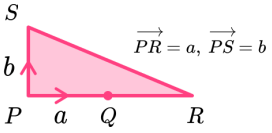
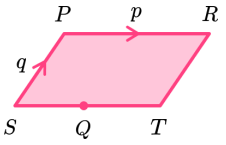
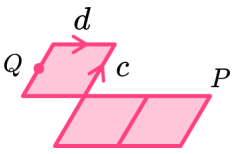
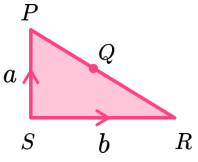
(3)

(6 marks)

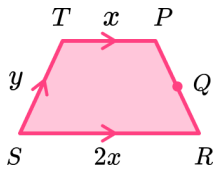
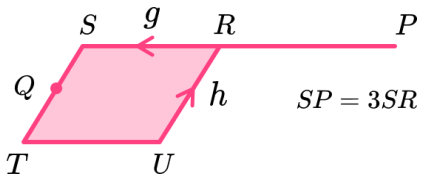
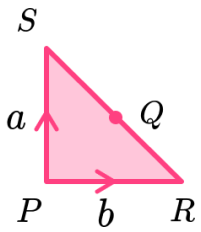
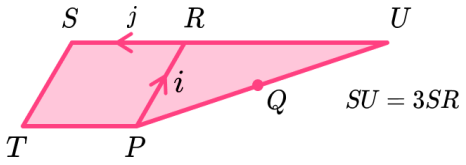
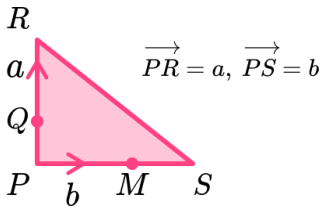
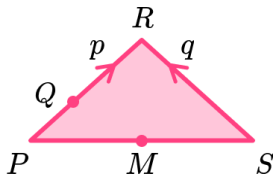
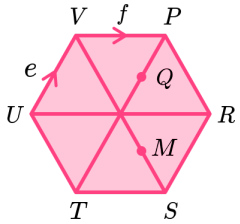
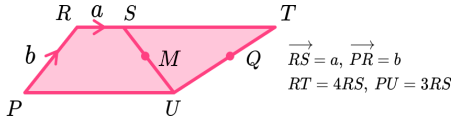
Vector problems - Answers

	Question	Answer
	Skill Questions	
Group A	In each question find the vector $\overrightarrow{AB}$. All diagrams are not to scale:	
	1) 	**1)** $-c + 2d$
	2) 	**2)** $3a - 2b$
	3) 	**3)** $2e - f$
	4) 	**4)** $-6a + 4b$
	5) 	**5)** $2a$
	6) 	**6)** $\frac{3}{2}a$
	7) 	**7)** $3p$
	8) 	**8)** $-2a$

Vector problems - Answers

Group A contd	**9)** 	**9)** $-a + b + 2c$
	10) 	**10)** $2m - 3n$
	11) 	**11)** $2a - 2b$
	12) 	**12)** $3.5a - 4b$
Group B	In questions 1-8 find the vector $\overrightarrow{PQ}$. In questions 9-12 find the vector $\overrightarrow{MQ}$. Both M and Q are midpoints. All diagrams are not to scale: **1)** 	**1)** $\frac{1}{2}a$
	2) 	**2)** $\frac{1}{2}p - q$
	3) 	**3)** $\frac{1}{2}c - 3d$
	4) 	**4)** $-\frac{1}{2}a + \frac{1}{2}b$

Vector problems - Answers

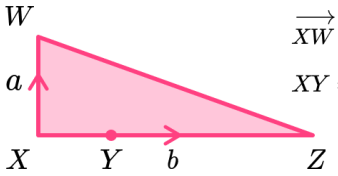
Group B contd	**5)** 	**5)** $\frac{1}{2}x - \frac{1}{2}y$
	6) 	**6)** $-3g - \frac{1}{2}h$
	7) 	**7)** $\frac{1}{2}a + \frac{1}{2}b$
	8) 	**8)** $\frac{1}{2}i - j$
	9) 	**9)** $\frac{1}{2}a - \frac{1}{2}b$
	10) 	**10)** $\frac{1}{2}q$
	11) 	**11)** $e - \frac{1}{2}f$
	12) 	**12)** $\frac{3}{2}a$

Vector problems - Answers

Group C

In each question find the vector $\overrightarrow{XY}$. All diagrams are not to scale:

1)

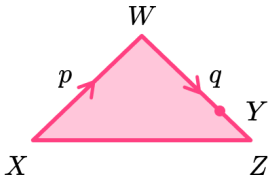


$\overrightarrow{XW} = a$, $\overrightarrow{XZ} = b$

$XY = \frac{1}{3}XZ$

1) $\frac{1}{3}b$

2)

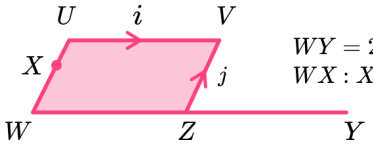


$\overrightarrow{XW} = p$, $\overrightarrow{WZ} = q$

$WY : YZ = 3 : 1$

2) $p + \frac{3}{4}q$

3) UVZW is a parallelogram

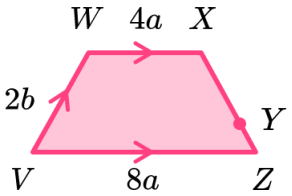


$WY = 2WZ$

$WX : XU = 2 : 1$

3) $2i - \frac{2}{3}j$

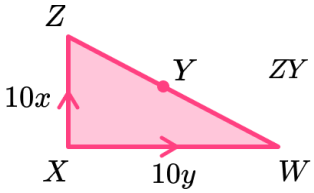
4)



$YZ = \frac{1}{4}XZ$

4) $2a - \frac{3}{2}b$

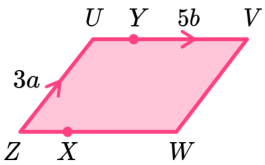
5)



$ZY : YW = 2 : 3$

5) $6x + 4y$

6) UVWZ is a parallelogram



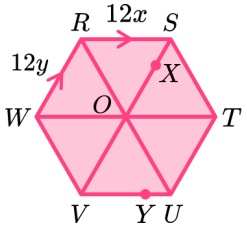
$\overrightarrow{ZU} = 3a$, $\overrightarrow{UV} = 5b$

$ZX = \frac{2}{3}XW$

$UY : YV = 1 : 4$

6) $3a - b$

7)

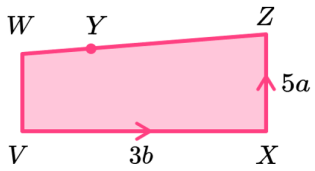
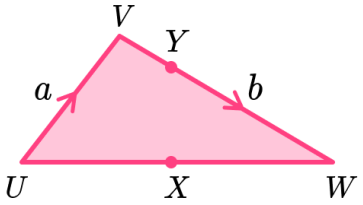
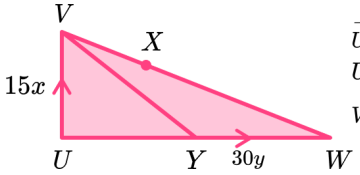
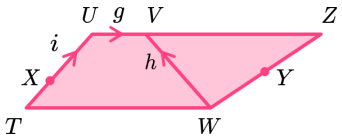
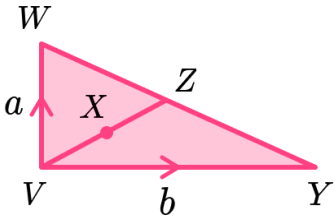


$SX = \frac{1}{4}SO$

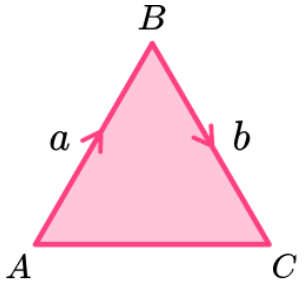
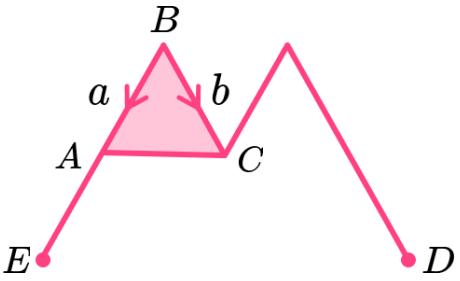
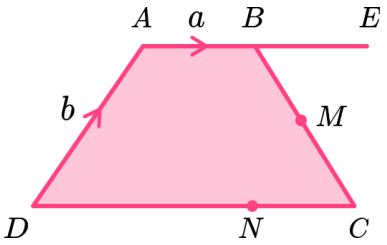
$VY : YU = 3 : 1$

7) $9x - 21y$

Vector problems - Answers

Group C contd	**8)**  $VW = \frac{1}{2}XZ$ $WY : YZ = 1 : 2$	**8)** $\frac{10}{3}a$ - $2b$
	9)  $X = midpoint\ UW$ $VY = \frac{1}{4}VW$	**9)** $\frac{1}{2}a - \frac{1}{4}b$
	10)  $\overrightarrow{UV} = 15x$, $\overrightarrow{UW} = 30y$ $UY : YW = 2 : 3$ $VX = \frac{1}{3}VW$	**10)** $-10x + 2y$
	11)  $\overrightarrow{UV} = g$, $\overrightarrow{VW} = h$, $\overrightarrow{TU} = i$ $UZ = 4UV$ $TX : XU = 1 : 3$ $Y = midpoint\ WZ$	**11)** $\frac{3}{4}i + \frac{5}{2}g - \frac{1}{2}h$
	12)  $WZ : ZY = 3 : 2$ $X = midpoint\ VZ$	**12)** $-\frac{1}{5}a + \frac{7}{10}b$

Vector problems - Answers

	Question	Answer
	Applied Questions	
1)	Point D is located using the vector $\overrightarrow{CD} = \boldsymbol{a} + 2\boldsymbol{b}$. Point E is located using the vector $\overrightarrow{AE} = -\boldsymbol{a}$.  Diagram **not** to scale **a)** Make a copy of the diagram, showing the location of the points D and E. **b)** Hence or otherwise find the vector $\overrightarrow{DE}$.	**a)**  **b)** $-3a - 3b$
2)	In the diagram below, $\overrightarrow{AB} = \boldsymbol{a}$, $\overrightarrow{DA} = \boldsymbol{b}$ and $\overrightarrow{DC} = 3\overrightarrow{AB}$. The line AB is extended so that $\overrightarrow{AE} = 2\overrightarrow{AB}$. M is the midpoint of BC. N is the point such that $DN : NC = 2 : 1$.  Diagram **not** to scale **a)** **i)** Find the vector $\overrightarrow{CD}$. **ii)** Find the vector $\overrightarrow{CB}$. **iii)** Find the vector $\overrightarrow{NM}$. **b)** Show NME is a straight line.	**a)** **i)** $\overrightarrow{CD} = -3\boldsymbol{a}$ **ii)** $\overrightarrow{CB} = \boldsymbol{b} - 2\boldsymbol{a}$ **iii)** $\overrightarrow{NM} = 0.5\boldsymbol{b}$ **b)** $\overrightarrow{ME} = 0.5\boldsymbol{b}$ so $\overrightarrow{NM} = \overrightarrow{ME}$. As the vectors share the same point M, NME is a straight line.

Vector problems - Answers

3)

In the diagram below, $\overrightarrow{AD} = \boldsymbol{x}$ and $\overrightarrow{DC} = \boldsymbol{y}$.
E is the midpoint of AC.

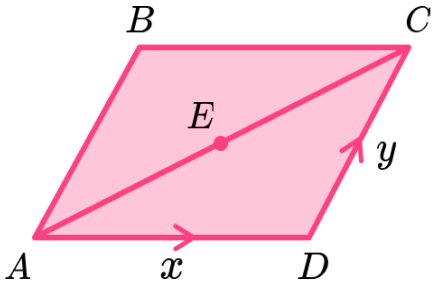


Diagram **not** to scale

a) Find the vector $\overrightarrow{AE}$.

b) Show that E is the midpoint of BD.

a) $\frac{1}{2}\boldsymbol{x} + \frac{1}{2}\boldsymbol{y}$

b) $\overrightarrow{BE} = -\boldsymbol{y} + \frac{1}{2}\boldsymbol{x} + \frac{1}{2}\boldsymbol{y} = \frac{1}{2}\boldsymbol{x} - \frac{1}{2}\boldsymbol{y}$

$\overrightarrow{ED} = -\frac{1}{2}\boldsymbol{x} - \frac{1}{2}\boldsymbol{y} + \boldsymbol{x} = \frac{1}{2}\boldsymbol{x} - \frac{1}{2}\boldsymbol{y}$

The two vectors are equal, therefore E is the midpoint

4)

In the diagram below, $\overrightarrow{BA} = 3\boldsymbol{a}$, $\overrightarrow{BC} = 3\boldsymbol{b}$ and $\overrightarrow{AF} = 3\overrightarrow{AD}$.
E is a point such that $\overrightarrow{ED} = n\boldsymbol{a}$.

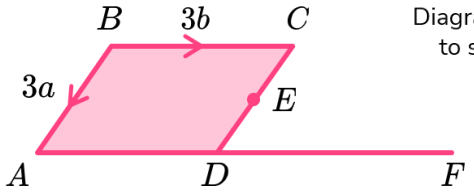
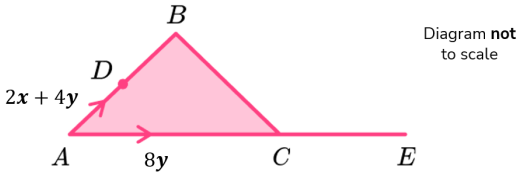
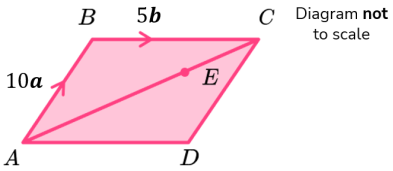
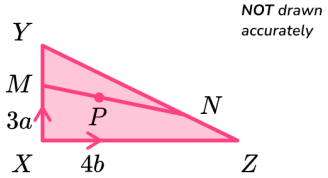


Diagram **not** to scale

Given that BEF is a straight line, find the value of n.

$\overrightarrow{BE} = 3\boldsymbol{b} + (3-n)\boldsymbol{a}$
$\overrightarrow{EF} = n\boldsymbol{a} + 6\boldsymbol{b}$
$2(3-n) = n$
$n = 2$

Vector problems - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	$\overrightarrow{AB} = 2\boldsymbol{x} + 4\boldsymbol{y}$, $\overrightarrow{AC} = 8\boldsymbol{y}$. D is the midpoint of AB. The line AC is extended so that $\overrightarrow{AE} = 1.5\overrightarrow{AC}$.  Diagram **not** to scale		
(a)	Find the vector $\overrightarrow{BC}$.	**(a)** $\overrightarrow{BC} = -2\boldsymbol{x} - 4\boldsymbol{y} + 8\boldsymbol{y} = -2\boldsymbol{x} + 4\boldsymbol{y}$	**(1)**
(b)	Find the vector $\overrightarrow{DE}$.	**(b)** $\overrightarrow{AE} = 12\boldsymbol{y}$, $\overrightarrow{DA} = -\boldsymbol{x} - 2\boldsymbol{y}$ $\overrightarrow{DE} = -\boldsymbol{x} - 2\boldsymbol{y} + 12\boldsymbol{y} = -\boldsymbol{x} + 10\boldsymbol{y}$	**(1)** **(1)**
2)	$\overrightarrow{AB} = 10\boldsymbol{a}$, $\overrightarrow{BC} = 5\boldsymbol{b}$. The point E lies on the line AC such that $AE : EC = 3 : 2$.  Diagram **not** to scale		
(a)	Find the vector $\overrightarrow{AC}$.	**(a)** $\overrightarrow{AC} = 10\boldsymbol{a} + 5\boldsymbol{b}$	**(1)**
(b)	Show that $\overrightarrow{AE} = k(2\boldsymbol{a} + \boldsymbol{b})$ and hence determine the value of k.	**(b)** $\overrightarrow{AE} = \frac{3}{5}(10\boldsymbol{a} + 5\boldsymbol{b})$ $= 6a + 3b = 3(2a + b)$ $k = 3$	**(1)** **(1)**
3)	$\overrightarrow{XY} = 3\boldsymbol{a}$, $\overrightarrow{XZ} = 4\boldsymbol{b}$ $\overrightarrow{MY} = \frac{1}{3}\overrightarrow{XY}$ $ZN : NY = 1 : 3$ P is the midpoint of $\overrightarrow{MN}$.  *NOT drawn accurately*		
(a)	Find the vector $\overrightarrow{MN}$.	**(a)** $\overrightarrow{MY} = \boldsymbol{a}$ $\overrightarrow{YN} = \frac{3}{4}(-3\boldsymbol{a} + 4\boldsymbol{b}) = -\frac{9}{4}\boldsymbol{a} + 3\boldsymbol{b}$ $\overrightarrow{MN} = \boldsymbol{a} - \frac{9}{4}\boldsymbol{a} + 3\boldsymbol{b} = -\frac{5}{4}\boldsymbol{a} + 3\boldsymbol{b}$	**(1)** **(1)** **(1)**

Vector problems - Mark Scheme

(b)	Find the vector $\overrightarrow{XP}$.	**(b)** $\overrightarrow{XM} = 2\boldsymbol{a}$ $\overrightarrow{MP} = -\frac{5}{8}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b}$ $\overrightarrow{XP} = 2\boldsymbol{a} - \frac{5}{8}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b} = \frac{11}{8}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b}$	**(1)** **(1)** **(1)**