

Composite and Inverse Functions - Worksheet

Skill

Group A - Evaluating composite functions

If $f(x) = 3x$, $g(x) = x^2$ and $h(x) = x + 1$, evaluate the composite functions for the given values.

1) $gf(1)$

2) $fg(1)$

3) $hf(1)$

4) $fh(3)$

5) $gh(3)$

6) $hg(3)$

7) $fgh(-2)$

8) $fhg(-2)$

9) $ghf(-2)$

Group B - Finding composite functions

If $f(x) = x + 3$, $g(x) = x^2$ and $h(x) = 2x$, find the expressions of these composite functions.

1) $fg(x)$

2) $gf(x)$

3) $hf(x)$

4) $fh(x)$

5) $gh(x)$

6) $hg(x)$

7) $fgh(x)$

8) $fhg(x)$

9) $gf h(x)$

Group C - Finding inverse functions

Find the inverse of these functions.

1) $f^{-1}(x)$ when $f(x) = x + 4$

2) $g^{-1}(x)$ when $g(x) = 3x$

3) $h^{-1}(x)$ when $h(x) = 6x + 5$

4) $f^{-1}(x)$ when $f(x) = \frac{x}{4} - 3$

5) $g^{-1}(x)$ when $g(x) = \frac{2x}{5} + 1$

6) $h^{-1}(x)$ when $h(x) = \frac{3(x-7)}{2}$

7) $f^{-1}(x)$ when $f(x) = \sqrt{x} + 5$

8) $g^{-1}(x)$ when $g(x) = \sqrt[3]{x - 2}$

9) $h^{-1}(x)$ when $h(x) = 4x^3 + 7$

Composite and Inverse Functions - Worksheet

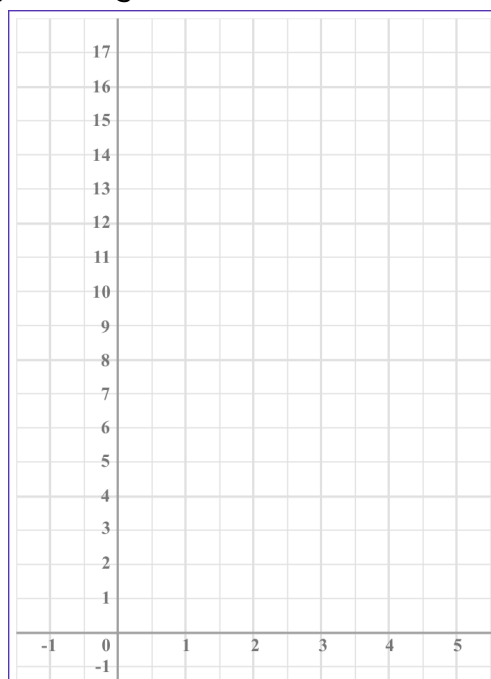
Applied

1) $f(x) = x - 3$ and $g(x) = x^2$

(a) Complete the table of values for $gf(x)$.

x	- 1	0	1	2	3	4	5
$gf(x)$		9			0	1	

(b) Plot the graph of $gf(x)$ on the grid below.



2) (a) Can you give an expression for a function $f(x)$ where $f(x) = f^{-1}(x)$?

(b) If $gh(x) = x$, what can you say about the relationship between the functions $h(x)$ and $g(x)$?

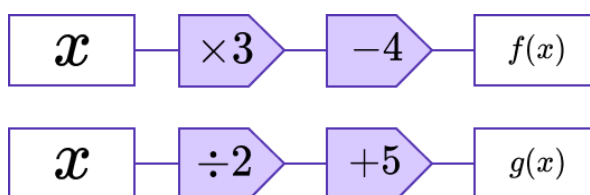
Composite and Inverse Functions - Worksheet

3) $f(x) = \sqrt{x} - 2$ is valid for values of $x > 0$. $g(x) = x + 6$ and $h(x) = x^2$.

(a) Find $f^{-1}(x)$.

(b) Solve when $f^{-1}(x) = hg(-9)$.

4) Here are two function machines showing the functions $f(x)$ and $g(x)$.



(a) Write down the expressions for $f^{-1}(x)$ and $g^{-1}(x)$.

(b) Write down the expression for $fgf(x)$ in the form $\frac{ax+b}{c}$.

Composite and Inverse Functions - Exam Questions

1) $f(x) = x^2$ and $g(x) = \frac{x-40}{3}$

(a) Find $fg(31)$
(2)

(b) Find the values of x when $f(x) = g^{-1}(x)$
(4)
(6 marks)

2) $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

(a) Find $fg(x)$
(2)

(b) Find when $fg(x) = gf(x)$
(4)
(6 marks)

3) $f(x) = \frac{x+5}{x+3}$

Find $f^{-1}(x)$
(4 marks)

4) $g(x) = 2x + 7$ and $h(x) = x(x + 3)$

(a) Find an expression for $hg(x)$
(3)

(b) Find the values of x when $h(x) = g^{-1}(15)$
(4)
(7 marks)

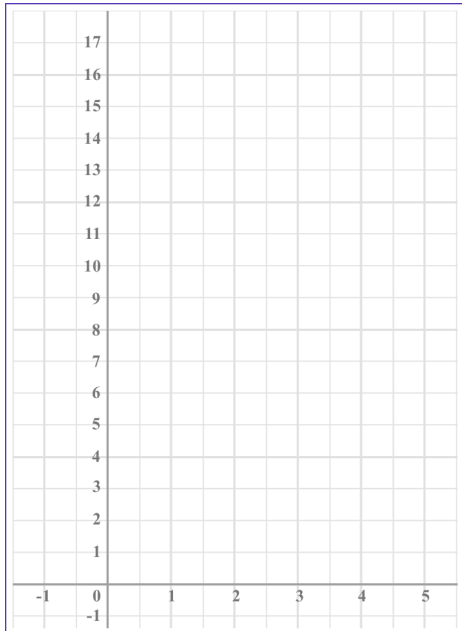
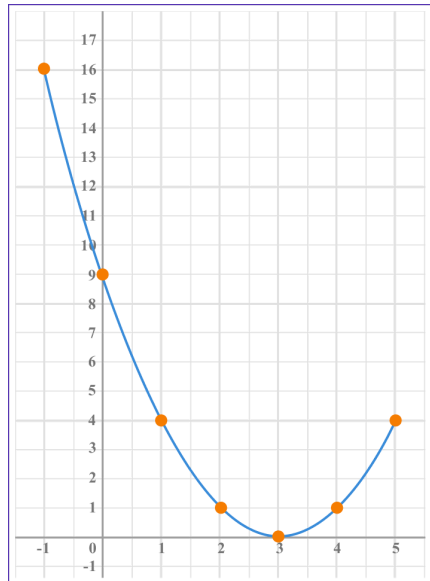
Composite and Inverse Functions - Answers

	Question	Answer
	Skill Questions	
Group A	If $f(x) = 3x$, $g(x) = x^2$ and $h(x) = x + 1$. Evaluate the composite functions for the given values. 1) $gf(1)$ 2) $fg(1)$ 3) $hf(1)$ 4) $fh(3)$ 5) $gh(3)$ 6) $hg(3)$ 7) $fgh(-2)$ 8) $fhg(-2)$ 9) $ghf(-2)$	1) 9 2) 3 3) 4 4) 12 5) 16 6) 10 7) 3 8) 15 9) 25
Group B	If $f(x) = x + 3$, $g(x) = x^2$ and $h(x) = 2x$. Find the expressions of these composite functions. 1) $fg(x)$ 2) $gf(x)$ 3) $hf(x)$ 4) $fh(x)$ 5) $gh(x)$ 6) $hg(x)$ 7) $fgh(x)$ 8) $fhg(x)$ 9) $gfh(x)$	1) $x^2 + 3$ 2) $x^2 + 6x + 9$ 3) $2x + 6$ 4) $2x + 3$ 5) $4x^2$ 6) $2x^2$ 7) $4x^2 + 3$ 8) $2x^2 + 3$ 9) $4x^2 + 12x + 9$

Composite and Inverse Functions - Answers

Group C	Find the inverse of these functions. 1) $f^{-1}(x)$ when $f(x) = x + 4$ 2) $g^{-1}(x)$ when $g(x) = 3x$ 3) $h^{-1}(x)$ when $h(x) = 6x + 5$ 4) $f^{-1}(x)$ when $f(x) = \frac{x}{4} - 3$ 5) $g^{-1}(x)$ when $g(x) = \frac{2x}{5} + 1$ 6) $h^{-1}(x)$ when $h(x) = \frac{3(x-7)}{2}$ 7) $f^{-1}(x)$ when $f(x) = \sqrt{x} + 5$ 8) $g^{-1}(x)$ when $g(x) = \sqrt[3]{x} - 2$ 9) $h^{-1}(x)$ when $h(x) = 4x^3 + 7$	1) $x - 4$ 2) $\frac{x}{3}$ 3) $\frac{x-5}{6}$ 4) $4x + 12$ 5) $\frac{5x-5}{2}$ 6) $\frac{2x}{3} + 7$ 7) $x^2 - 10x + 25$ 8) $x^3 + 2$ 9) $\sqrt[3]{\frac{x-7}{4}}$
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Composite and Inverse Functions - Answers

	Question	Answer																
	Applied Questions																	
1)	$f(x) = x - 3$ and $g(x) = x^2$ (a) Complete the table of values for $gf(x)$. <table border="1"><tr><td>x</td><td>- 1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$gf(x)$</td><td></td><td>9</td><td></td><td></td><td>0</td><td>1</td><td></td></tr></table> (b) Plot the graph of $gf(x)$ on the grid below. 	x	- 1	0	1	2	3	4	5	$gf(x)$		9			0	1		(a) 16, 4, 1, 4 (b) 
x	- 1	0	1	2	3	4	5											
$gf(x)$		9			0	1												
2)	(a) Can you give an expression for a function $f(x)$ where $f(x) = f^{-1}(x)$? (b) If $gh(x) = x$, what can you say about the relationship between the functions $h(x)$ and $g(x)$?	(a) $f(x) = 4 - x$ (b) They will be inverses of each other.																

Composite and Inverse Functions - Answers

3)	$f(x) = \sqrt{x} - 2$ is valid for values of $x > 0$. $g(x) = x + 6$ and $h(x) = x^2$. (a) Find $f^{-1}(x)$. (b) Solve when $f^{-1}(x) = hg(-9)$.	(a) $x^2 + 4x + 4$ (b) $x^2 + 4x + 4 = 9$ $x^2 + 4x - 5 = 0$ $(x + 5)(x - 1) = 0$ $x = 1, \text{ cannot be } -5$
4)	Here are two function machines showing the functions $f(x)$ and $g(x)$. (a) Write down the expressions for $f^{-1}(x)$ and $g^{-1}(x)$. (b) Write down the expression for $fgf(x)$ in the form $\frac{ax+b}{c}$.	(a) $f^{-1}(x) = \frac{x+4}{3},$ $g^{-1}(x) = 2x - 10$ (b) $\frac{9x+10}{2}$

Composite and Inverse Functions - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	$f(x) = x^2$ and $g(x) = \frac{x-40}{3}$		
(a)	Find $fg(31)$.	(a) $\frac{31-40}{3}$ 9	(2)
(b)	Find the values of x when $f(x) = g^{-1}(x)$.	(b) $g^{-1}(x) = 3x + 40$ Set equal $x^2 = 3x + 40$ Set equal to 0 and factorise $(x - 8)(x + 5) = 0$ $x = 8, -5$	(4)
2)	$f(x) = 2x + 1$ and $g(x) = x^2 - 2$.		
(a)	Find $fg(x)$.	(a) $2(x^2 - 2) + 1$ $2x^2 - 3$	(2)
(b)	Find when $fg(x) = gf(x)$.	(b) $gf(x) = (2x + 1)^2 - 2$ Set equal $2x^2 - 3 = 4x^2 + 4x - 1$ Attempt to solve/ factorise $2(x + 1)(x + 1) = 0$ (1) $x = -1(1)$	(4)
3)	$f(x) = \frac{x+5}{x+3}$		
	Find $f^{-1}(x)$.	$x = \frac{y+5}{y+3}$ $x(y + 3) = y + 5$ Multiplying by denominator $xy - y = 5 - 3x$ Expand and isolate $y(x - 1) = 5 - 3x$ y factored out $f^{-1}(x) = \frac{5-3x}{x-1}$	(4)

Composite and Inverse Functions - Mark Scheme

4)	$g(x) = 2x + 7$ and $h(x) = x(x + 3)$		
(a)	Find an expression for $hg(x)$.	(a) Substituted $(2x + 7)(2x + 7 + 3)$ or equivalent $(2x + 7)^2 + 3(2x + 7)$ Expanded terms $4x^2 + 20x + 14x + 70$ Terms collected $4x^2 + 34x + 70$	(3)
(b)	Find the values of x when $h(x) = g^{-1}(15)$.	(b) $g^{-1}(15) = 4$ found Set equal $x(x + 3) = 4$ Factorised $(x + 4)(x - 1) = 0$ $x = -4, 1$	(4)

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