

Completing the Square - Worksheet

Skill

Group A - Even coefficients of x

Complete the square.

1) $x^2 + 2x + 1$

2) $x^2 - 2x + 1$

3) $x^2 + 4x + 4$

4) $x^2 + 2x + 5$

5) $x^2 - 2x + 5$

6) $x^2 + 4x + 8$

7) $x^2 + 2x - 2$

8) $x^2 - 2x - 2$

9) $x^2 + 4x + 1$

10) $x^2 + 6x + 9$

11) $x^2 + 16 + 6x$

12) $16 + x^2 - 6x$

Group B - (Mostly) odd coefficients of x

Complete the square.

1) $x^2 + 8x + 16$

2) $x^2 + 8x + 28$

3) $x^2 - 8x + 28$

4) $x^2 + 3x + \frac{9}{4}$

5) $x^2 - 3x + \frac{9}{4}$

6) $x^2 + 5x + \frac{25}{4}$

7) $x^2 + 3x + 5$

8) $x^2 + 3x + 6$

9) $x^2 + 3x + 7$

10) $5x + 2 + x^2$

11) $2 + x^2 - 5x$

12) $x^2 + 7x + 2$

Group C - Coefficient of $x^2 \neq 1$

Complete the square.

1) $1x^2 + 2x + 6$

2) $2x^2 + 4x + 12$

3) $2x^2 - 4x + 12$

4) $3x^2 - 6x + 4$

5) $3x^2 - 6x + 5$

6) $5 + 3x^2 - 12x$

7) $3x^2 - 9x + 4$

8) $4x + 5x^2 - 2$

9) $5x^2 - 4x - 2$

10) $-x^2 + 12x - 16$

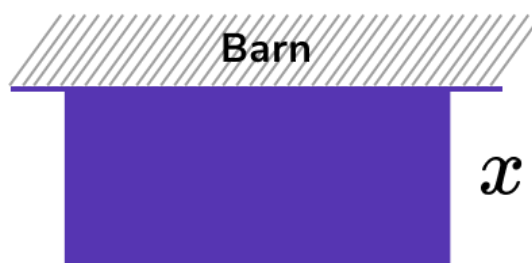
11) $12x - 2x^2 - 16$

12) $-3x^2 + 12x - 5$

Completing the Square - Worksheet

Applied

- 1) (a) By completing the square, solve the equation $x^2 - 4x - 10 = 0$. Leave your answer in surd form.
- (b) By completing the square, solve the equation $2x^2 + 5x - 3 = 0$. What do you notice about your answer?
- 2) A farmer wants to build an animal enclosure against one wall of his barn as shown in the diagram. He has 200m of fencing.



- (a) Given the width of the enclosure is x metres, write an expression for the length of the enclosure, and hence find an expression for the area.
- (b) By completing the square, find the maximum area of the enclosure, and give the length and width of the fencing needed.
- 3) (a) Complete the square for the expression $x^2 - 4x + 5$.
- (b) Find the coordinates of the turning point of the graph of $y = x^2 - 4x + 5$ and hence sketch the graph.
- (c) What happens if you try to solve the quadratic equation $x^2 - 4x + 5 = 0$? Use your graph to help you to explain your answer.
- 4) Complete the square on the equation $ax^2 + bx + c = 0$ to obtain the quadratic formula.

Completing the Square - Exam Questions

- 1) Express $x^2 + 6x + 20$ in the form $(x + a)^2 + b$, where a and b are integers.

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(2 marks)

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- 2) (a) Express $x^2 - 6x + 4$ in the form $(x + a)^2 + b$, where a and b are integers.

.....
(2)

- (b) Hence, or otherwise, find the coordinates of the turning point of the graph of $y = x^2 - 6x + 4$.

.....
(1)
(3 marks)

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- 3) By completing the square, solve $x^2 - 8x - 5 = 0$, giving your answers in surd form.

.....
(4 marks)

Completing the Square - Exam Questions

- 4) (a) $2x^2 + 12x - 8$ can be written in the form $a(x + b)^2 - c$ where a , b and c are positive integers. Work out the values of a , b and c .

.....
(3)

- (b) Hence, or otherwise, solve the equation $2x^2 + 12x - 8 = 0$.
Give your answers in surd form.

.....
(2)
(5 marks)

-
- 5) (a) Write $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a , b and c are integers.

.....
(3)

- (b) Hence, or otherwise, find the coordinates of the turning point of the graph of $y = 3x^2 - 6x + 5$.

.....
(1)
(4 marks)

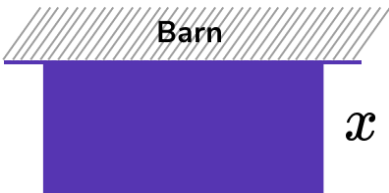
Completing the Square - Answers

	Question	Answer
	Skill Questions	
Group A	Complete the square. 1) $x^2 + 2x + 1$ 2) $x^2 - 2x + 1$ 3) $x^2 + 4x + 4$ 4) $x^2 + 2x + 5$ 5) $x^2 - 2x + 5$ 6) $x^2 + 4x + 8$ 7) $x^2 + 2x - 2$ 8) $x^2 - 2x - 2$ 9) $x^2 + 4x + 1$ 10) $x^2 + 6x + 9$ 11) $x^2 + 16 + 6x$ 12) $16 + x^2 - 6x$	1) $(x + 1)^2$ 2) $(x - 1)^2$ 3) $(x + 2)^2$ 4) $(x + 1)^2 + 4$ 5) $(x - 1)^2 + 4$ 6) $(x + 2)^2 + 4$ 7) $(x + 1)^2 - 3$ 8) $(x - 1)^2 - 3$ 9) $(x + 2)^2 - 3$ 10) $(x + 3)^2$ 11) $(x + 3)^2 + 7$ 12) $(x - 3)^2 + 7$
Group B	Complete the square. 1) $x^2 + 8x + 16$ 2) $x^2 + 8x + 28$ 3) $x^2 - 8x + 28$ 4) $x^2 + 3x + \frac{9}{4}$ 5) $x^2 - 3x + \frac{9}{4}$ 6) $x^2 + 5x + \frac{25}{4}$ 7) $x^2 + 3x + 5$	1) $(x + 4)^2$ 2) $(x + 4)^2 + 12$ 3) $(x - 4)^2 + 12$ 4) $(x + \frac{3}{2})^2$ 5) $(x - \frac{3}{2})^2$ 6) $(x + \frac{5}{2})^2$ 7) $(x + \frac{3}{2})^2 + \frac{11}{4}$

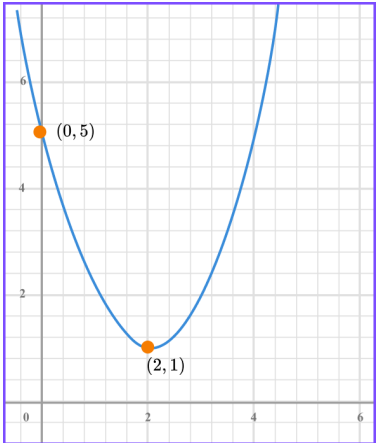
Completing the Square - Answers

Group B contd	8) $x^2 + 3x + 6$ 9) $x^2 + 3x + 7$ 10) $5x + 2 + x^2$ 11) $2 + x^2 - 5x$ 12) $x^2 + 7x + 2$	8) $(x + \frac{3}{2})^2 + \frac{15}{4}$ 9) $(x + \frac{3}{2})^2 + \frac{19}{4}$ 10) $(x + \frac{5}{2})^2 - \frac{17}{4}$ 11) $(x - \frac{5}{2})^2 - \frac{17}{4}$ 12) $(x + \frac{7}{2})^2 - \frac{41}{4}$
Group C	Complete the square. 1) $1x^2 + 2x + 6$ 2) $2x^2 + 4x + 12$ 3) $2x^2 - 4x + 12$ 4) $3x^2 - 6x + 4$ 5) $3x^2 - 6x + 5$ 6) $5 + 3x^2 - 12x$ 7) $3x^2 - 9x + 4$ 8) $4x + 5x^2 - 2$ 9) $5x^2 - 4x - 2$ 10) $-x^2 + 12x - 16$ 11) $12x - 2x^2 - 16$ 12) $-3x^2 + 4x - 5$	1) $(x + 1)^2 + 5$ 2) $2(x + 1)^2 + 10$ 3) $2(x - 1)^2 + 10$ 4) $3(x - 1)^2 + 1$ 5) $3(x - 1)^2 + 2$ 6) $3(x - 2)^2 - 7$ 7) $3(x - \frac{3}{2})^2 - \frac{11}{4}$ 8) $5(x + \frac{2}{5})^2 - \frac{14}{5}$ 9) $5(x - \frac{2}{5})^2 - \frac{14}{5}$ 10) $-(x - 6)^2 + 20$ 11) $-2(x - 3)^2 + 2$ 12) $-3(x - \frac{2}{3})^2 - \frac{11}{3}$

Completing the Square - Answers

	Question	Answer
	Applied Questions	
1)	(a) By completing the square, solve the equation $x^2 - 4x - 10 = 0$. Leave your answer in surd form. (b) By completing the square, solve the equation $2x^2 + 5x - 3 = 0$. What do you notice about your answer?	(a) CTS: $(x - 2)^2 - 14$ $x = 2 \pm \sqrt{14}$ (b) CTS: $2(x + \frac{5}{4})^2 - \frac{49}{8}$ $x = -\frac{1}{2}, x = -3$ The solutions do not contain surds - this equation could also be solved by factorising.
2)	A farmer wants to build an animal enclosure against one wall of his barn as shown in the diagram. He has 200m of fencing.  (a) Given the width of the enclosure is x metres, write an expression for the length of the enclosure, and hence find an expression for the area. (b) By completing the square, find the maximum area of the enclosure, and give the length and width of the fencing needed.	(a) Length: $200 - 2x$ Area: $200x - 2x^2$ CTS: $-2(x - 50)^2 + 5000$ (b) Max area: $5000m^2$ Occurs when $x = 50$ Length: $100m$ Width: $50m$

Completing the Square - Answers

3)	(a) Complete the square for the expression $x^2 - 4x + 5$. (b) Find the coordinates of the turning point of the graph of $y = x^2 - 4x + 5$ and hence sketch the graph. (c) What happens if you try to solve the quadratic equation $x^2 - 4x + 5 = 0$? Use your graph to help you to explain your answer.	(a) $(x - 2)^2 + 1$ (b) Coordinates  (c) There are no (real) solutions - graphically, the curve does not intersect the x axis.
4)	By completing the square on the general quadratic: $ax^2 + bx + c = 0$, obtain the quadratic formula.	Remove factor of a to allow us to CTS: $a\left[x^2 + \frac{b}{a}\right] + c = 0$ CTS within the square brackets: $a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c = 0$ Make x the subject: $a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] = -c$ $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$ $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$ $x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Completing the Square - Mark Scheme

	Question	Answer	
	Exam Questions		
1)	Express $x^2 + 6x + 20$ in the form $(x + a)^2 + b$, where a and b are integers.	$(x + 3)^2$ seen $(x + 3)^2 + 11$	(1) (1)
2) (a)	Express $x^2 - 6x + 4$ in the form $(x + a)^2 + b$, where a and b are integers.	(a) $(x - 3)^2$ seen $(x - 3)^2 - 5$	(1) (1)
(b)	Hence, or otherwise, find the coordinates of the turning point of the graph of $y = x^2 - 6x + 4$.	(b) $(3, - 5)$	(1)
3)	By completing the square, solve $x^2 - 8x - 5 = 0$, giving your answers in surd form.	$(x - 4)^2$ $(x - 4)^2 - 21 = 0$ oe $x - 4 = \pm \sqrt{21}$ $x = 4 + \sqrt{21}$ and $x = 4 - \sqrt{21}$	(1) (1) (1) (1)
4) (a)	$2x^2 + 12x - 8$ can be written in the form $a(x + b)^2 - c$ where a , b and c are positive integers. Work out the values of a , b and c .	(a) $(x + 3)^2$ $2(x + 3)^2 - 18 - 8$ oe $a = 2, b = 3, c = - 26$ or embedded as $2(x + 3)^2 - 26$	(1) (1) (1)
(b)	Hence, or otherwise, solve the equation $2x^2 + 12x - 8 = 0$. Give your answers in surd form.	(b) $2(x + 3)^2 - 26 = 0$ $x + 3 = \pm \sqrt{13}$ $x = - 3 + \sqrt{13}$ and $x = - 3 - \sqrt{13}$	(1) (1) (1)

Completing the Square - Mark Scheme

5) (a)	Write $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a , b and c are integers.	(a) $(x - 1)^2$ $3(x - 1)^2 - 3 + 5$ oe $3(x - 1)^2 + 2$	(1) (1) (1)
(b)	Hence, or otherwise, find the coordinates of the turning point of the graph of $y = 3x^2 - 6x + 5$.	(b) $(1, 2)$	(1)

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